Phys. Status Solidi B **252**, No. 7, 1559–1564 (2015) / **DOI** 10.1002/pssb.201552218

physica status solidi b
www.pss-b.com
basic solid state physics

Non-porous grooved single-material auxetics

Luke Mizzi[1], Ruben Gatt[1], and Joseph N. Grima[*,1,2]

[1] Metamaterials Unit, Faculty of Science, University of Malta, Msida MSD2080, Malta
[2] Department of Chemistry, Faculty of Science, University of Malta, Msida MSD2080, Malta

Received 16 April 2015, revised 3 June 2015, accepted 8 June 2015
Published online 1 July 2015

Keywords auxetic, finite element analysis, mechanical metamaterials, mechanical properties, Poisson's ratio, rotating rigid units

* Corresponding author: e-mail auxetic@um.edu.mt, Phone +356 23402274

Over the years, great advances have been made in the field of auxetic metamaterials where one of the main focuses was the production of systems which can be produced through simple and relatively inexpensive means. In this work, auxetic systems created through the introduction of elliptical grooves, meant to mimic the rotating units mechanism, are proposed and analyzed. These systems were found to have the potential to exhibit Poisson's ratios ranging from *ca.*-1 to the Poisson's ratio of the material of the system itself, with some systems also possessing the remarkable property of a zero Poisson's ratio. The final product is a non-porous metamaterial with potentially tailor-made Poisson's ratio properties.

1 Introduction Auxetics are systems which exhibit the counterintuitive property of possessing a negative Poisson's ratio [1]. This property, which arises primarily from the geometry and deformation mechanism of the system, has been shown to exist on a wide number of scales, ranging from the macro- to the nano-level [2–11]. A number of geometries and mechanisms which have the potential to give rise to this anomalous characteristic have also been discovered and devised. One such mechanism is the rotating rigid units mechanism which typically involves an array of polygonal or solid units that rotate relative to each upon application of strain.

Numerous studies have been conducted on these systems, particularly with respect to the effect that shape and rigidity of the rotating unit have on the mechanical properties of these systems [12–18] (see Fig. 1). Recent studies by our group have also examined the effect of hierarchy on these units [19], and how this effect can be implemented though perforations, slits, or cuts [20–22]. These studies have confirmed that these systems have the potential to exhibit a wide range of Poisson's ratios, ranging from highly positive to giant negative values.

Although this class of auxetic structures is amongst the most oft studied systems with respect to its potential to exhibit a negative Poisson's ratio, there are still a number of *lacunae* within the field which have yet to be explored. For example, while various studies have investigated 2D and 3D perforated structures based on these rotating systems [19–30], as well as composite systems [31], none of these works have considered systems where a groove rather than a complete perforation is used to create an auxetic system from a previously non-auxetic sheet or block of material. Such a system is expected to have the added advantage of obtaining an auxetic material without the creation of pores or holes within the system and thus could be extremely useful for applications where a non-porous auxetic material is required.

In view of this, in this work, the effect of the introduction of grooves into a block of material, designed in manner so as to create a system which mimics the deformation behavior of the rotating square mechanism (a system which possesses an isotropic Poisson's ratio of -1 in its ideal state), will be investigated. In particular, this work will focus on the effect which the depth of the groove has on the mechanical properties of the system, as well as the role played by the pore size and shape of the system.

2 Methodology In this work, a series of rotating square-like systems produced through the introduction of elliptically-shaped grooves within the system will be investigated through Finite Element modelling using the Ansys13 software. These systems were simulated using the SOLID187 element, a higher order 3D, 10-node

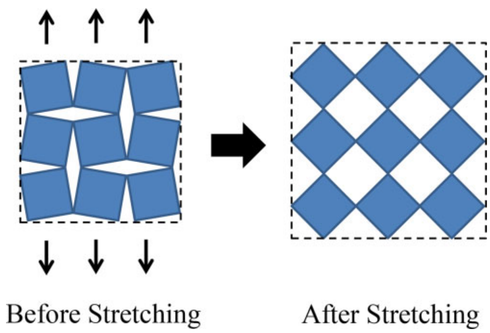

Figure 1 A diagram showing the deformation of a rotating square system under the effect of tensile strain.

element with quadratic displacement behavior, which possesses three degrees of freedom at each node, i.e., x, y, and z translations. The meshing of the system was created using the smart sizing option provided by the software, which was set to its finest setting and each structure was solved linearly. This setting ensures that the system has extra mesh refinement at its vertices and edges, i.e., the regions which are expected to bear the brunt of the structure's deformation. This method is more computationally efficient than using a uniformly fine mesh, which typically results in a system with an unnecessarily large number of elements in regions which do not deform. The material properties of structural steel were used to simulate these systems, i.e., a Poisson's ratio of 0.3 and Young's Modulus of 200 GPa were used. In order to minimize computational time, a representative volume unitcell with periodic boundary conditions in the x- and y-directions were used to simulate these systems. As shown in Fig. 2, the parameters used to define the system are the total depth/thickness of the system, d, the depth of the grooves, l, the vertical and horizontal dimensions of the elliptical grooves, r_1 and r_2, respectively, and the separation between the grooves, s. Since no periodic boundary conditions were applied in the z-direction, the infinite system is also assumed to have a total thickness or depth of d.

In total, simulations on 360 grooved structures were conducted where, referring to Fig. 2, the parameter d and r_1 were kept constant at 4 and 1 m, respectively, while the parameters r_2, s, and l of these systems were altered as follows:

$$-\log\left(\frac{h}{d}\right) = -\log\left(\frac{d-2l}{d}\right) \in 0,\ 0.05,\ldots,1.95,\quad r_2 \in 2,\ 3,\ 4\text{m},\quad s \in 0.1,\ 0.3,\ 0.5\text{m}, \tag{1}$$

where a value of $-\log(h/d) = 0$ would correspond to a solid block of material and $-\log(h/d) = 1.95$ corresponds to a system with very deep grooves where a very thin sheet, equivalent to *ca.* 1% of the whole thickness, remains between the grooves. Note that the variations to the l parameter are based on a geometric scale of the l/d ratio. This scale was chosen in order to increase the density of data in the region where $2l/d$ is close to 1, i.e., when the system is nearly fully perforated, since the strongest gradients in the subsequent plots were expected to be observed in this region. In addition, while the length in z-direction for all structures was fixed at d, the x- and y-dimensions of each system change according to r_1, r_2, and s.

Additionally, so as to enable a proper comparison with the equivalent fully perforated systems, nine additional simulations were performed with $l = 2, r_2 \in 2,\ 3,\ 4$ and $s \in 0.1,\ 0.3,\ 0.5$. These systems would correspond to $-\log[(d-2l)/d] \to\infty$.

Each of the 369 system so constructed was subjected to a uniaxial tensile strain ε_x in the x-direction of 0.1. Using the resulting strain ε_y in the y-direction, which was calculated by averaging the displacement of the nodes in the topmost edges of the structure and dividing it by the original length of the system, the Poisson's ratio ν_{xy} was calculated as

$$\nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x}. \tag{2}$$

The stress for loading in the x-direction, σ_x, was also calculated by measuring the resulting forces acting on the nodes on rightmost edges and dividing the total force by the cross-sectional area of the unit cell perpendicular to the x-direction, i.e., the yz-cross-section. The Young's modulus in the x-direction was then calculated from

$$E_x = \frac{\sigma_x}{\varepsilon_x}. \tag{3}$$

Note that since all systems studied exhibit a rotational symmetry of order 4, the mechanical properties in the x and y-directions can be assumed to be identical, i.e., $\nu_{xy} = \nu_{yx}$ and $E_x = E_y$.

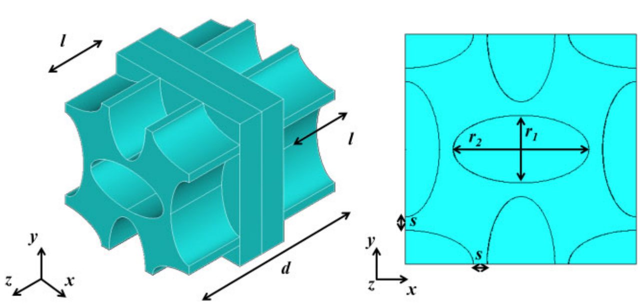

Figure 2 Diagrams showing the parameters used to describe the systems modelled here. The parameter h indicates the thickness of the central block of material and is equal to $d-2l$.

3 Results and discussion Plots showing the variations of the Poisson's ratio, V_{xy}, and the Young's modulus, E_x, with changes to the groove depth to thickness ratio are shown in Fig. 3.

As one can observe, in all systems, the Poisson's ratio decreases as the depth of the elliptical grooves increases

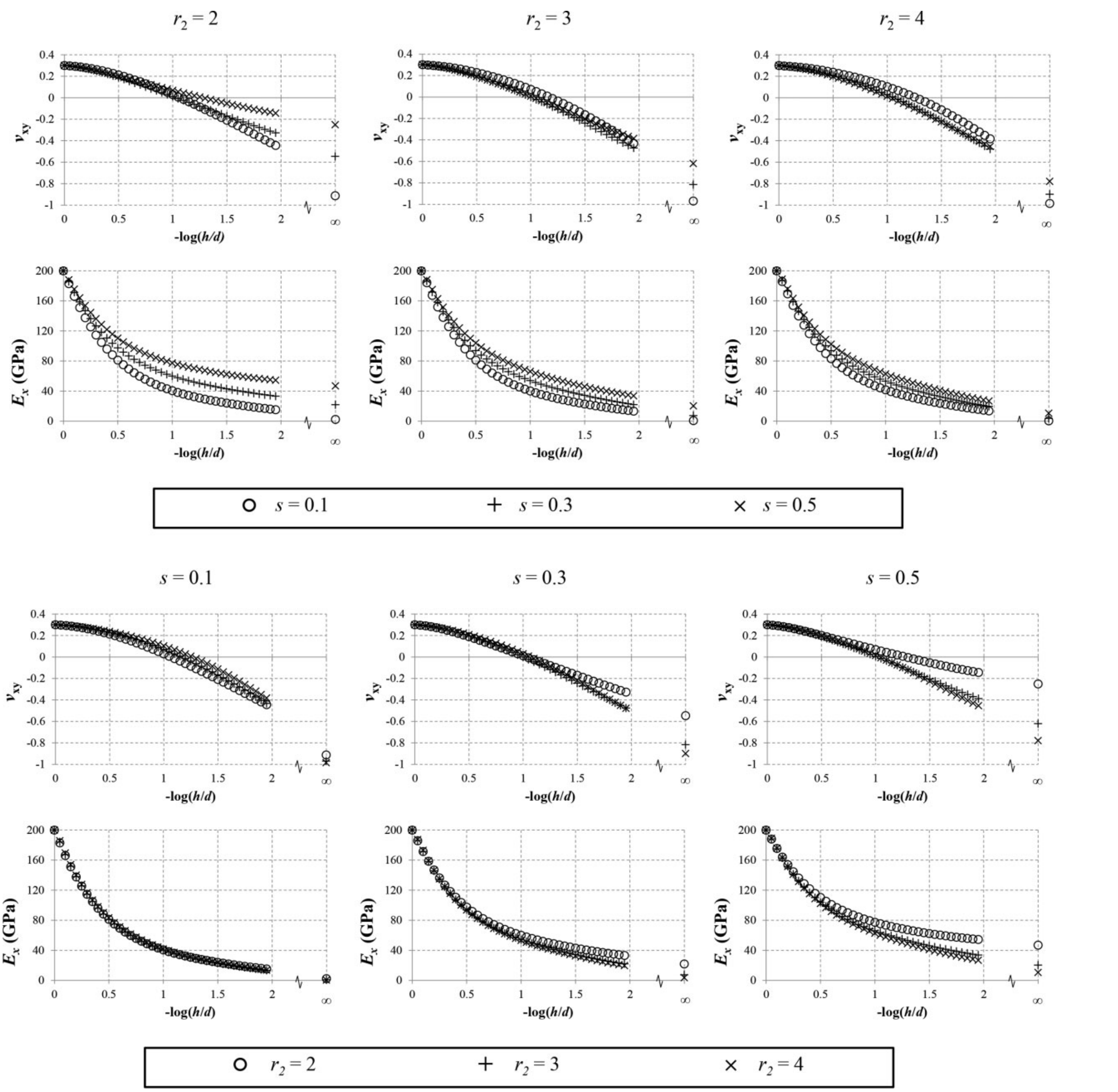

Figure 3 Plots showing the variation of the Poisson's ratio and Young's modulus with increasing groove depth for systems with different s values, while keeping r_2 constant (top panel) and with different r_2 values while keeping s constant (bottom panel). Note that a value of 0 on the x-axis denotes a block of material without grooves while a value of ∞ represents a fully perforated system.

until it reaches its lowest value, which is negative for all systems, when the groove becomes a complete perforation. On the other hand, when the groove becomes shallower, the Poisson's ratio tends toward 0.3, i.e., the Poisson's ratio of the material of the system, as expected since the system now effectively becomes a solid block of material. In between these two extremes, there is a whole spectrum of Poisson's ratios which changes from positive to zero to negative, meaning that through this setup it is possible to create a non-porous single-material auxetic system made from conventional positive Poisson's ratio material through a relatively easy and inexpensive method, i.e., through the introduction of grooves. However, probably even more important, is the fact that these grooves can act as a very simple, yet effective manner for fine-tuning the Poisson's ratio so as to produce a final product with a tailor-made macroscopic Poisson's ratio.

Similar trends can also be observed for the Young's modulus, which is highest when the groove are not present (i.e., system is a whole block of material) and lowest when the groove becomes a perforation. This suggests that the Young's moduli are also tunable, although not in an

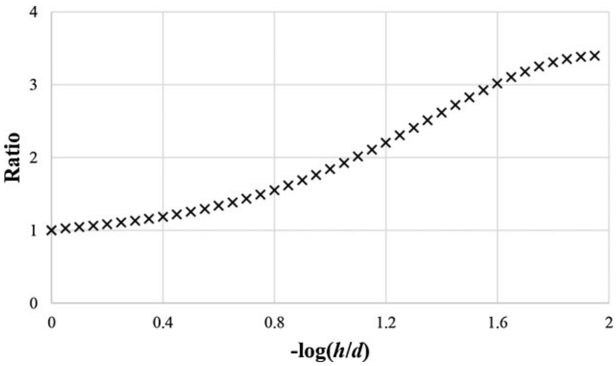

Figure 4 Plot of E_s/E_x^* against $-\log(h/d)$ for a system where $s = 0.1$ and $r_2 = 2$.

independent manner from the Poisson's ratios, since in all case, the most auxetic systems are also the least stiff, i.e., though this design, one cannot increase the auxeticity and at the same time increase the stiffness. Here it should also be noted that the Young's modulus of each of the grooved systems considered here is consistently higher than the equivalent system made from the equivalent individual components, i.e., two perforated sheets of thickness l and a solid sheet of thickness h, which are stretched in parallel which would be expected to have a hypothetical Young's modulus given by

$$E_x^* = \frac{2l}{d} E_{x,\infty} + \frac{h}{d} E_{x,0}. \tag{4}$$

This is clearly illustrated in Fig. 4, which plots the ratio E_s/E_x^* for the systems where $s = 0.1$ and $r_2 = 2$ which show that the Young's modulus of the grooved structures can be up to *ca.* 3.5 times higher than the hypothetical systems containing individual components.

Before proceeding any further it would be useful to look more closely on the way that these systems deform upon the application of uniaxial stress. As evident from Fig. 5, uniaxial stretching in the horizontal x-direction results in a deformation where, to a first approximation, the parts of the system which are meant to resemble the "rigid squares" rotate relative to each other hence resulting in the observed lowering of the Poisson's ratio, and in some cases even auxeticity. In fact, had the groove been extended to become a complete perforation, the system would behave in a very similar manner to what is described in earlier work [23, 25], i.e., mimic well the behavior of the idealized rotating squares model. However, the presence of the solid block of material in the center of the structure hinders this rotation, with the result that there is an uneven extent of rotations along the depth of the structure, with the topmost/lowermost regions rotating the most and the innermost regions, which are effectively "semi-clamped" by the solid block in the middle, rotating the least. This effectively results in what may be described as a torsional deformation of these units.

This mode of deformations may explain the trends in the mechanical properties. For example, the stiffening described above may be explained by the inability of the system to deform as a result of the "semi-clamping," an effect which is consistent with the work by Lim and others who also reported that a semi-auxetic laminate or structures made from alternate layers of positive/negative Poisson's ratio components is expected to exhibit enhanced stiffness [32, 33]. Similar arguments based on this "semi-clamping" effect by the stiffer central region on the whole system may be made to explain the observed trends in the Poisson's ratio on varying the depth of the groove.

As shown in Fig. 3 above, variations to the parameters r_1 and r_2 also have an effect on the mechanical properties of the system, especially as the depth of the groove and the separation between the grooves increases. In fact, as one can

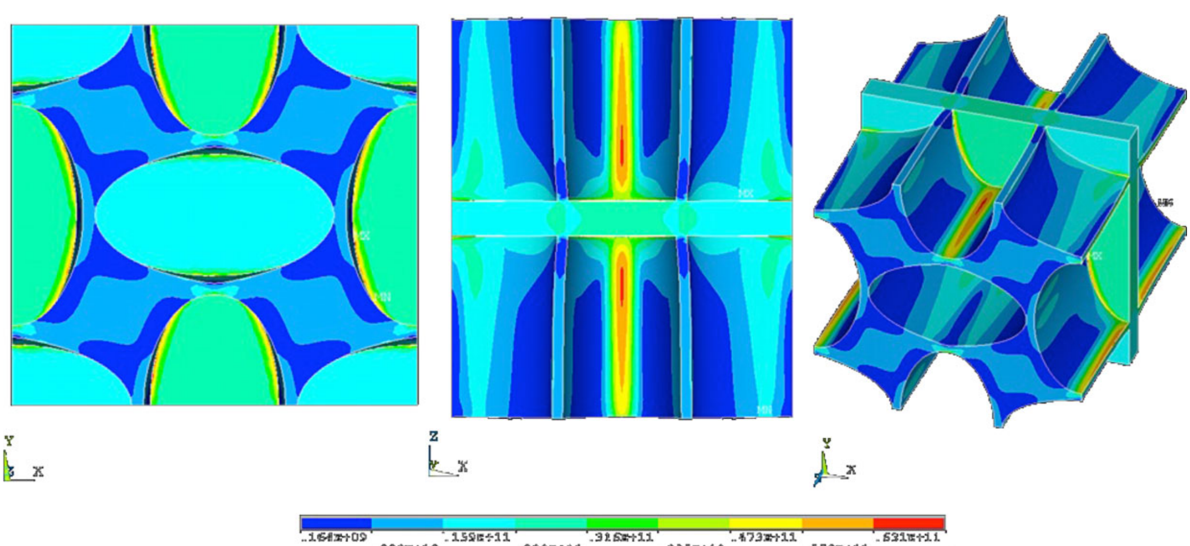

Figure 5 Diagram showing variations in stress intensity in a system which exhibits a Poisson's ratio of approximately zero.

observe from the plots in Fig. 3, for systems with large separation ($s = 0.5$), the Young's modulus and Poisson's ratio decrease as the r_2/r_1 ratio increases. This trend is also evident, although less obviously, in systems with intermediate separation ($s = 0.3$), before disappearing completely for systems with $s = 0.1$. This indicates that the separation between the elliptical grooves plays a significant role in determining the effect which groove shape has on the mechanical properties of the system.

The effect of separation on the Young's modulus and Poisson's ratio of these systems is also not as straightforward as it is for perforated systems (see Fig. 3). Normally, in auxetic perforated systems, an increase in s results in an increase of both the Poisson's ratio and the Young's modulus of the system since on increasing this parameter one is also increasing the deviation from the ideal rotating squares model [20, 25]. Although this trend is retained for the Young's modulus, it is not, however, observed for the Poisson's ratio in systems with large r_2 values.

A particular point worthy of mention is the fact that there is a certain point in every plot in Fig. 3, where the Poisson's ratio of the system becomes zero. This property, which is unique in itself, comes about as a result of the positive Poisson's ratio of the central block of material in the center of the structure and the negative Poisson's ratio of the upper and lower parts of the structure (which is borne as a result to the geometry formed through the introduction of grooves) completely cancelling each other out, and is extremely useful of its own accord as it is usually accompanied by several derived characteristics such as a natural ability to form tubular curvatures without dog-boning-like edge effects [5]. This effect is clearly manifested if one had to attempt to make a cylinder from a zero Poisson's ratio system since this property has been shown to eliminate anticlastic curvature [32].

Before we conclude, it is important to highlight that the work conducted here is only a pilot study on the auxetic potential of these systems. Further work is necessary before one can progress to the next step, which is the actual manufacture and testing of these systems, particularly with respect to the high strain behavior of these structures, as well as the dependence of the macroscopic properties of the system on the intrinsic mechanical properties of the constituent material. Both of these factors are expected to have a significant effect on the overall mechanical properties of the system. Furthermore, one should also look into the manner in which stresses are distributed within the structure, so as to identify where failure is more likely to occur. In this case, as evident form the images in Fig. 4 which show how the stress is distributed in a particular structure having *ca.* zero Poisson's ratio, the weaker regions include those which roughly correspond the "hinges" in the idealized rotating squares system.

Also, given the number of possible variables in the system studied here, a further more focused study could be performed where, for example, one would keep the density of the system constant while changing the other parameters, or keeping the amount of material used to construct system as constant and studying the influence of the shape of the elliptical grooves on the properties of the system while maintaining constant the volume of used material. Obviously, one may also attempt to use other shapes apart from ellipses to produce the grooves, provided that the resulting system can still mimic the behavior of auxetic mechanisms. For example, the elliptical grooves used in this study could easily be replaced by stars so as to produce a system which resembles a rotating triangles system.

Apart from changing the geometric variables, one could also investigate the effect that the constituent materials have on the macroscopic properties. In particular, the methodology employed here can also be used to study systems where the material properties of the sheet wedged between the rotating square geometries above and below it are changed and/or *vice-versa*. This work can also be a blueprint for future studies on composite systems based on the same geometric model.

Such further studies could result in the design of novel systems based on the concept presented here which have macroscopic properties that are tailor made for specific practical applications. This is extremely important, especially for applications where a non-porous auxetic or zero Poisson's ratio material is required, such as in the manufacture of tubes or pressure vessels [34].

4 Conclusions In this study, the mechanical properties of systems produced through the introduction of grooves designed to produce a rotating square-like system were investigated using a Finite Element approach. It was found that a significant number of these systems possess the potential to exhibit near zero or a negative Poisson's ratio despite the fact that no pores were introduced within the material. Given the simplicity of these systems and the expected relatively low cost of manufacture, it is hoped that this work will be of use to researchers and industrialists looking to create non-porous, single material auxetic metamaterials.

Acknowledgements This work has been funded through the Malta Council for Science and Technology through the R&I-2012-061 Project (SMESH). The authors would like to thank the University of Malta and the partner involved in this project, Mater Dei Hospital.

References

[1] K. E. Evans, M. A. Nkansah, I. J. Hutchinson, and S. C. Rogers, Nature **253**, 124–124 (1991).
[2] J. N. Grima, S. Winczewski, L. Mizzi, M. C. Grech, R. Cauchi, R. Gatt, D. Attard, K. W. Wojciechowski, and J. Rybicki, Adv. Mater. **27**, 1455–1459 (2015).
[3] F. Scarpa, P. Pastorino, A. Garelli, S. Patsias, and M. Ruzzene, Phys. Status Solidi B **242**, 681–694 (2005).
[4] K. W. Wojciechowski, Phys. Lett. A **137**, 60–64 (1989).
[5] L. Mizzi, D. Attard, A. Casha, J. N. Grima, and R. Gatt, Phys. Status Solidi B **251**, 328–337 (2014).
[6] R. Gatt, J. P. Brincat, K. M. Azzopardi, L. Mizzi, and J. N. Grima, Adv. Eng. Mater. **17**, 189–198 (2015).

[7] A. Alderson, P. J. Davies, M. R. Williams, K. E. Evans, K. L. Alderson, and J. N. Grima, Mol. Simul. **31**, 897–905 (2005).
[8] T. Buckmann, R. Schittny, M. Thiel, M. Kadic, G. W. Milton, and M. Wegener, New J. Phys. **16**, 033032 (2014).
[9] G. Cicala, G. Recca, L. Oliveri, Y. Perikleous, F. Scarpa, C. Lira, A. Lorato, D. J. Grube, and G. Ziegmann, Compos. Struct. **94**, 3556–3562 (2012).
[10] R. S. Lakes, Science **235**, 1038–1040 (1987).
[11] G. Milton, J. Mech. Phys. Solids **40**, 1105–1137 (1992).
[12] J. N. Grima and K. E. Evans, J. Mater. Sci. Lett. **19**, 1563–1565 (2000).
[13] Y. Ishibashi and M. Iwata, J. Phys. Soc. Jpn. **69**, 2702–2703 (2000).
[14] A. Alderson and K. E. Evans, Phys. Rev. Lett. **89**, 225503 (2002).
[15] J. N. Grima, A. Alderson, and K. E. Evans, Comput. Methods Sci. Technol. **10**, 137–145 (2004).
[16] J. N. Grima, R. Gatt, B. Ellul, and E. Chetcuti, J. Non-Cryst. Solids **354**, 4214–4220 (2008).
[17] D. Attard, E. Manicaro, and J. N. Grima, Phys. Status Solidi B **246**, 2033–2044 (2009).
[18] R. Gatt, R. Caruana-Gauci, D. Attard, A. R. Casha, W. Wolak, K. Dudek, L. Mizzi, and J. N. Grima, Phys. Status Solidi B **251**, 321–327 (2014).
[19] R. Gatt, L. Mizzi, J. I. Azzopardi, K. M. Azzopardi, D. Attard, A. Casha, J. Briffa, and J. N. Grima, Sci. Rep. **5**, 8395 (2015).
[20] J. N. Grima and R. Gatt, Adv. Eng. Mater. **12**, 460–464 (2010).
[21] J. N. Grima, R. Gatt, B. Ellul, and E. Chetcuti, J. Non-Cryst. Solids **356**(37–40), 1980–1987 (2010).
[22] L. Mizzi, K. Azzopardi, D. Attard, J. N. Grima, and R. Gatt, Phys. Status Solidi RRL **9**, 425 (2015), DOI: 10.1002/pssr.201510178.
[23] K. Bertoldi, M. C. Boyce, S. Deschanel, S. M. Prange, and T. Mullin, J. Mech. Phys. Solids **56**, 2642–2688 (2008).
[24] Y. Cho, J. H. Shin, A. Costa, T. A. Kim, V. Kunin, J. Li, S. Y. Lee, S. Yang, H. N. Han, I. S. Choi, and D. J. Srolovitz, Proc. Natl. Acad. Sci. USA **111**, 17390–17395 (2014).
[25] M. Taylor, L. Francesconi, M. Gerendas, A. Shanian, C. Carson, and K. Bertoldi, Adv. Mater. **26**, 2365–2370 (2014).
[26] S. Babaee, J. Shim, J. C. Weaver, E. R. Chen, N. Patel, and K. Bertoldi, Adv. Mater. **25**(36), 5044–5049 (2013).
[27] K. Bertoldi, P. M. Reis, S. Willshaw, and T. Mullin, Adv. Mater. **22**, 361–366 (2010).
[28] T. Mullin, S. Willshaw, and F. Box, Soft Mater. **9**, 4951 (2013).
[29] J. T. B. Overvelde, S. Shan, and K. Bertoldi, Adv. Mater. **24**(17), 2337–2342 (2012).
[30] K. W. Wojciechowski, Lecture presented during the 8th Workshop on Auxetics and Related Systems, Szczecin, 8th September 2011, Joint Conferences on Advanced Materials—Abstract Book (TASK Publishing, Gdansk, Poland, 2011), p. 59.
[31] T. Strek, H. Jopek, and M. Nienartowicz, Phys. Status Solidi B **252**, 1540–1550 (2015), this issue.
[32] J. N. Grima, L. Oliveri, D. Attard, B. Ellul, R. Gatt, G. Cicala, and G. Recca, Adv. Eng. Mater. **12**(9), 855–862 (2010).
[33] T. C. Lim, Eur. J. Mech. A, Solids **28**, 752–756 (2009).
[34] B. Ellul, M. Muscat, and J. N. Grima, Phys. Status Solidi B **246**(9), 2025–2032 (2009).