

# Auxetic metamaterials exhibiting giant negative Poisson's ratios

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Auxetic metamaterials have generated a great deal of interest in recent years. In this letter, a novel approach which can be used to produce such auxetic metamaterials is presented. This method involves the introduction of patterned slit perforations within a sheet/block of material in order to create systems

which resemble a large variety of auxetic systems ranging from rotating units to chiral honeycombs. These perforated systems have also been shown to have the potential to exhibit giant negative Poisson's ratios.

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**1 Introduction** Auxetics [1] are materials/systems which exhibit the anomalous property of a negative Poisson's ratio, i.e. they expand in at least one transverse direction when stretched in a longitudinal direction. This behaviour is normally derived from the geometry and deformation mechanism of the system and thus is normally referred to as being scale independent. In fact, numerous auxetic systems containing similar geometric motifs and mechanical behaviour have been discovered at a number of levels ranging from the macro- to the nano-scale [2–5]. These systems have generated a great deal of interest within the scientific community throughout the years due to their vast number of potential applications which range from biomedical devices such as stents to personal protection equipment [6, 7]. Some of these uses are derived from a number of additional characteristics which often accompany a negative Poisson's ratio, such as synclastic curvature [8], increased indentation resistance [9] and enhanced acoustic and vibration properties [10].

In recent years, there has been significant interest on the development of novel methods which can be employed to produce auxetic mechanical metamaterials [11], with minimal expense and time. One such method, which is becoming increasingly popular, involves the introduction of specific perforation patterns in a conventional or positive Poisson's ratio block/sheet of material [12–21]. In other words this involves cutting pieces of material from a block

or sheet, which results in a system possessing a geometric configuration which is similar to that observed in a number of systems with the potential to exhibit auxeticity. This technique has been used thus far to create auxetic perforated systems which resemble rotating structures based on squares, triangles and cubes amongst others.

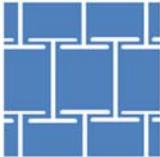
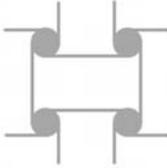
However, although there have been major advances in this field of auxetics, the full potential of these systems still remains untapped. Many of the aforementioned works, for example, consider primarily 'area' perforations with a limited number of patterns. These works, however, suffer from one major drawback, which is that in order to convert a non-auxetic material into an auxetic metamaterial there must be a loss of material. This shortcoming can be particularly debilitating in the case of applications where an expensive or limited amount of material is available for use such as in the case of skin grafting and thus puts a metaphorical lid on the vast potential of these systems for use in a number of fields. This disadvantage, however, is not insurmountable. In fact it may be eliminated completely if cuts are used to introduce slits rather than 'area' perforations within the system. Such slit-type perforations which mimic rotating systems have only been briefly explored thus far, with preliminary works focusing mainly on the most basic and least versatile auxetic systems and their hierarchical counterparts [12, 13, 19]. Recently, Shan et al. have also presented novel perforated slit perforated sys-

tems based on other rotating units [21]. The current state of the art, however, may be greatly expanded since, as amply demonstrated through rigorous analytical modelling, an extremely vast range of negative Poisson's ratios are attainable from a large range rotating unit models, the blueprints on which all existing perforated systems work [22–24].

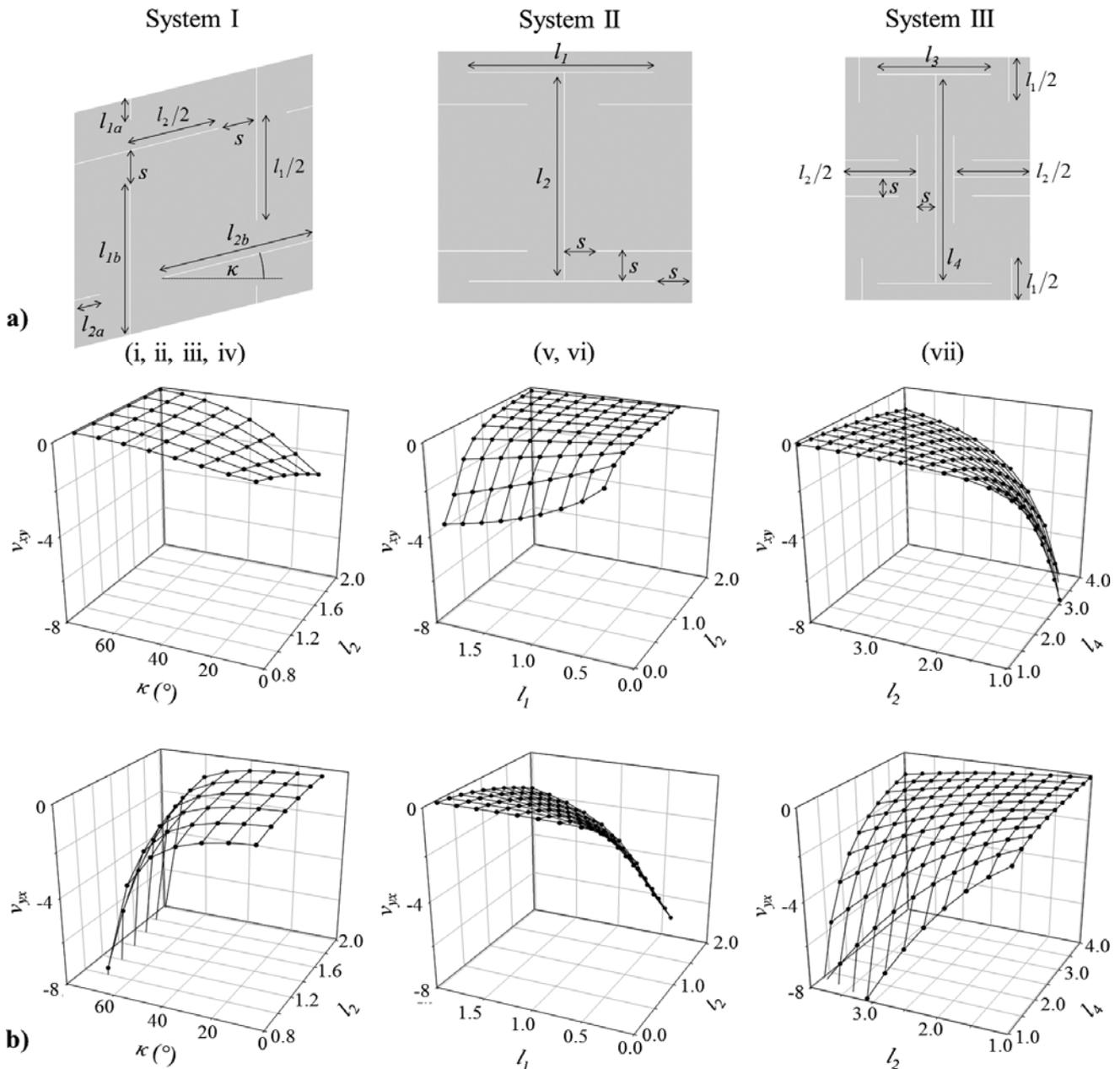
In view of all this, in this letter, a number of novel perforated systems created through slit-type perforations, i.e. perforations where a non-significant amount of material is lost, are being proposed and analysed. Besides eliminating material waste, these systems, which are specifically designed to exhibit deformation behaviour similar to a number of auxetic systems, are also expected to possess a greater stretching potential since in each case they represent the *quasi*-closed state of their respective idealised mechanism.

**2 Concept and design** A pictorial representation of the systems studied here and the auxetic systems they are designed to emulate is shown in Fig. 1. The first figure (Fig. 1(i)) shows the simplest of these designs, a series of alternating horizontal and vertical slits of equal length in-

serted in such a manner so as to mimic a rotating squares system. Similar systems attempting to duplicate this mechanism using area perforations have been previously suggested by a number of researchers using circular, elliptical and diamond perforations, with these systems achieving a relatively small range of negative Poisson's ratios [14–16]. However, in terms of material conservation and tensile strain performance, systems based on slits are expected to outperform these models since they more closely resemble the fully closed conformation of this structure [12, 21]. In addition, these previous studies, in the case of elliptical and diamond perforations, considered only structures where the two sets of alternating equally sized perforations were perpendicular to each other. On removing these two rigid conditions (i.e. equal size and perpendicular orientation of the slits), however, one can obtain a new class of systems which resemble rotating rectangles, rhombi and parallelograms, such as the ones shown in Fig. 1(ii–iv). These systems have been shown, through analytical modelling, to have the potential to exhibit a much vaster range of negative Poisson's ratios than rotating square systems, and thus it is expected that, in turn,

Straight Slits		I-shaped Slits	
Slit Pattern	Mechanism	Slit Pattern	Mechanism
(i) 	 Rotating Squares	(v) 	 'Fibrils and Nodes'
(ii) 	 Type I Rotating Rectangles	(vi) 	 Re-entrant Honeycomb
(iii) 	 Type $\alpha$ Rotating Rhombi	(vii) 	 Anti-tetrachiral
(iv) 	 Type $I\alpha$ Rotating Parallelograms		

**Figure 1** Pictorial representation of the variety of auxetic mechanisms that are expected to be derived from the respective slit perforation patterns.



**Figure 2** a) Diagrams showing the three systems and their respective parameters which can be used to design representative unit cells of all the systems shown in Fig. 1. Note that for every system based on System I:  $l_{1a} + l_{1b} = l_1$  and  $l_{2a} + l_{2b} = l_2$ . b) 3D-plots depicting the changes in  $v_{xy}$  and  $v_{jx}$  with various geometric parameters. Note that the Poisson's ratios of systems with values lower than  $-8$  are presented in the Supporting Information.

these perforated systems exhibit a much larger spectrum of mechanical properties than their rotating square-like counterparts.

The first system shown in Fig. 2(a) (hereby denoted as System I) encompasses all of these systems made from straight line slits; each of which can be obtained by varying the geometric parameters used to describe it. These parameters include the length of the vertical slit,  $l_1$ , the length of the inclined slit,  $l_2$ , the separation between the edge of one slit and the centre of another,  $s$ , and the tilt of the in-

clined slit,  $\kappa$ . Straight line slits, although considered to be the simplest, are not the only possible type of slit perforations which can be used to recreate auxetic mechanisms from a sheet of material.

As shown in Fig. 1(v–vii), another, slightly more complex slit perforation type can be used to produce mechanical metamaterials based on other auxetic mechanisms. Based on an ‘I-shaped’ slit pattern, these systems can be generated by altering the size, separation and orientation of these slit perforations. The systems shown in Fig. 1(v) and

(vi), which are expected to mimic the re-entrant mechanism found in 'fibrils and nodules' structures and re-entrant hexagonal honeycombs, respectively (depending on perforation dimensions), can be created if the central strips of all 'I'-shaped perforations are aligned parallel to each other. On the other hand, the system shown in Fig. 1(vii), which is expected to emulate the well-known anti-tetrachiral honeycomb mechanism, can be formed if the 'I'-shaped slits are introduced in an alternating-perpendicular conformation. As shown in Fig. 2(a), the former systems are defined by  $l_1$  and  $l_2$ , the length dimensions of the 'I'-shaped slit and  $s$ , the separation between the perforations (System II), while the latter are defined by the parameters of vertical and horizontally aligned 'I'-shaped slits,  $l_1$ ,  $l_2$ ,  $l_3$  and  $l_4$  and  $s$ , the separation (System III).

Together, the perforated systems presented here are expected to yield a vast range of negative Poisson's ratios where large tunability can be merged with the advantageous factors derived from using slit-type perforations.

**3 Methodology** The systems introduced in the previous section were studied through a Finite Element approach using the Ansys13 software. These systems were simulated using the PLANE183 element, a plane stress, higher order 2D 8-node or 6-node element with quadratic displacement behaviour. Following convergence tests, the mesh size of each system was set to half the thickness of the slit,  $t$ , which for practical reasons could not be infinitesimally thin and thus was set to the very small value. The material properties of rubber were used to simulate these systems and, in order to minimise computational time, each system was simulated as a representative unit cell (see Fig. 2(a)) with periodic boundary conditions in the  $x$ - and  $y$ -directions and solved linearly. Deformation was induced by the application of a uniaxial tensile force in both loading directions separately (see Supporting Information).

Table 1 shows the range of parameters used to design the systems studied in this work.

These dimensions were chosen on the basis that they would provide a complete picture of the most extreme and typical geometries which can be obtained for each system. The only systems from this set which were not considered were those whose geometric design resulted in overlapping slits, and hence fragmentation of the structure.

**Table 1** Parameters used to create the systems simulated here (see Fig. 2a). System I comprises the architectures shown in Fig. 1(i-iv), System II comprises the architectures shown in Fig. 1(v, vi), and System III includes the architecture shown in Fig. 1(vii).

System I	System II	System III
$l_1 = 1$	$l_1 \in 0.2 \dots 2, 0.2$	$l_1 = 0.7 \times l_4$
$l_2 \in 1 \dots 2.8, 0.2$	$l_2 \in 0.2 \dots 2, 0.2$	$l_2 \in 1 \dots 4, 0.3$
$\kappa \in 0 \dots 80, 10^\circ$	$s = 0.1$	$l_3 = 0.7 \times l_2$
$s = 0.1$		$l_4 \in 1 \dots 4, 0.3$
		$s = 0.1$

Following the initial set of simulations, additional non-linear simulations were conducted on the most promising structures in order to determine their high strain tensile behaviour (see Supporting Information).

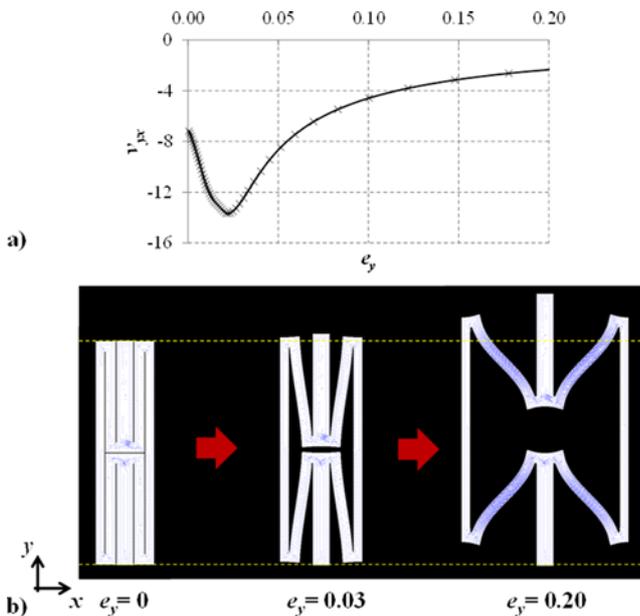
**4 Results and discussion** As shown in Fig. 2(b), all perforated systems simulated here exhibited auxetic behaviour, ranging from nearly zero to giant negative values. The most negative Poisson's ratios were found in systems where the two geometric parameters varied were at their opposite extremities, i.e. one very low and the other very high, which also resulted in highly anisotropic behaviour.

In the case of System I, when the tilt of the inclined perforation,  $\kappa$ , was  $0^\circ$ , and  $l_2$  was 1.0 (i.e. equal to  $l_1$ ), the system showed an isotropic Poisson's ratio of ca.  $-1$ , indicating that it is deforming via a mechanism which is extremely similar to the rotating square mechanism (see ANIM01.avi, provided as Supporting Information). Other systems where  $\kappa$  was also equal to zero, showed increasing anisotropy on increasing the discrepancy between  $l_1$  and  $l_2$ . This was due to the 'rotating units' adopting a more rectangular conformation and thus behaving in a similar manner to Type I rotating rectangles. Also, as expected, on increasing the tilt,  $\kappa$ , the perforated systems started behaving similarly to Type Ia rotating parallelograms and Type  $\alpha$  rhombi (the latter in the case where  $l_2$  was equal to 1.0) (see ANIM02/03/04.avi).

In comparison to System I, System II showed less variation with respect to its Poisson's ratios. However a significant number of systems still possessed giant negative Poisson's ratios with values lower than  $-4$ . The lowest values were observed in the  $v_{yx}$  plane for systems with small  $l_1$  and large  $l_2$  values. These systems deformed similarly to the re-entrant hexagonal honeycomb system. On the other hand, systems with large  $l_1$  and small  $l_2$  values resembled more closely the re-entrant 'fibril and nodule' mechanism, which resulted in Poisson's ratio close to 0 for the  $v_{yx}$  plane and large negative values for the  $v_{xy}$  plane (see ANIM05/06.avi).

System III proved to be the most versatile of all systems studied in this work. As shown in Fig. 2(b), it has the potential to exhibit Poisson's ratios ranging from nearly 0 to ca.  $-13$  (see Supporting Information). Similarly to the other two systems, System III exhibited giant negative Poisson's ratios when there was considerable difference in the lengths of the alternating slits,  $l_2$  and  $l_4$ . This may be explained by considering a previous study by Gatt et al. [25], where it was shown that when the ligament lengths are not equal, the anti-tetrachiral honeycomb does not retain its isotropic Poisson's ratio of  $-1$ , but becomes anisotropic, with the potential to exhibit a very wide range of negative Poisson's ratios (see ANIM07.avi).

The results obtained from the linear simulations, although extremely promising, can only give a glimpse of the real-life mechanical behaviour of these systems. Thus, in order to confirm that the giant Poisson's ratio obtained is not merely an artefact of the application of an infinitesi-



**Figure 3** a) Poisson's ratio (calculated from the differential strains) against engineering strain for a structure based on System II having parameters  $l_1 = 4$  and  $l_2 = 2$ . This method for the calculation of the Poisson's ratio has previously been employed to investigate the auxetic potential of graphene, which also showed a nonlinear response to stress [3]. b) Diagram of the same system at different degrees of strain in the  $y$ -direction.

mally small tensile strain but rather represents the actual mechanical properties of these systems, nonlinear simulations were conducted on two of the most promising systems studied here. As shown in Fig. 3(b), the re-entrant system based on System II actually exhibits an extremely large initial negative Poisson's ratio while the nonlinear simulations also indicate that the differential Poisson's ratio of the system actually becomes more negative with strains up to ca. 3%. This is followed by an increase on further stretching the system; however the differential Poisson's ratio still remains highly negative at large strains. ( $\nu_{yx} \approx -3$  at  $\epsilon_x = 0.20$ ). Further information on this can be found in the Supporting Information.

All this is very significant as this study has shown that it is possible to create perforated systems which exhibit a wide range of negative Poisson's ratio without necessitating any material losses. This is considered to be important from both an economic and a practical perspective since besides preventing material waste, it is arguably also considerably simpler to create a cut rather than an area perforation within a system, which would allow them to carve out niches for themselves within the field of auxetics.

One such niche is in the field of skin grafting. In a skin grafting operation, a piece of skin is cut from a healthy section of the body, perforated, stretched and transplanted onto a burnt or damaged region. Since the aim of surgery is to heal the patient, it is imperative that the removed skin is much smaller than the target damaged area. In order to in-

crease its surface coverage, the cut skin is perforated and stretched. If the skin graft was to be perforated in such a manner as described in this work, one may expect improved results in comparison with the current perforation pattern being used, which results in a conventional (positive Poisson's ratio) material response. Moreover, since in skin grafting operations the graft is perforated with cuts using a dermatome (a type of mesher), using any of these perforation patterns would not necessitate a complete change in the meshing procedure.

Another possible application for these systems is for use in blast curtains. Auxetic blast curtains have already been shown by previous researchers to be superior to non-auxetic ones [26]. These curtains are typically produced using auxetic yarn, however through the method employed here, one can produce them using materials with a positive Poisson's ratio. Moreover, the fact that slit perforated systems have a negligible pore size in their undeformed state is an added advantage over their 'area' perforated counterparts since the increase in pore size upon impact with the blast wave is expected to be considerably lower.

Before concluding, it is important to note that the work conducted here is merely a pilot study on the potential of this class of auxetic structures. Further work is required in order to evaluate the full potential of these systems, particularly with respect to their behaviour at extremely high strain conditions and the role which their material properties and out-of-plane thickness play on their mechanical behaviour. However, the concept presented here still yielded some extremely promising results, particularly in the case of the nonlinear simulations, and gives an insightful view of the deformation behaviour of these systems. Moreover, the fact that such a vast range of negative Poisson's ratios was obtained from merely two slit-type perforations is indicative of the enormous potential which this class of perforated systems has in the field of mechanical metamaterials, since the same techniques employed in this study may be reproduced using alternative slit-type perforation shapes and patterns to achieve even more diverse mechanical properties.

**5 Conclusion** In this study, a novel class of mechanical metamaterials created through the introduction of slit-type perforations, were proposed and investigated with respect to their potential to exhibit a negative Poisson's ratio. Each system studied here showed the ability to exhibit this property and since these systems do not involve material losses and can be created through techniques which are inadmissible for use on area-type perforations, it is expected that they could be useful for a number of potential specialized applications of auxetic systems such as skin grafting. Given the relative simplicity of these systems and vast tunability with respect to their Poisson's ratio, it is hoped that this work will be of significant aid in the attempts of researchers and industrialists alike in their quest to create new auxetic metamaterials.

**Supporting Information** Additional supporting information may be found in the online version of this article at the publisher's website.

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