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Smart metamaterials with tunable auxetic and other properties

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Abstract

Auxetic and other mechanical metamaterials are typically studied in situations where they are subjected solely to mechanical forces or displacements even though they may be designed to exhibit additional anomalous behaviour or tunability when subjected to other disturbances such as changes in temperature or magnetic fields. It is shown that externally applied magnetic fields can tune the geometry and macroscopic properties of known auxetics that incorporate magnetic component/s, thus resulting in a change of their macroscopic properties. Anomalous properties which are observed in such novel magneto-mechanical systems include tunable Poisson's ratios, bi-stability or multi-stability, depending on the applied magnetic fields, and other electromagnetic–mechanical effects such as strain dependent induced electric currents and magnetic fields. The properties exhibited depend, amongst other things, on the relative position and orientation of the magnetic insertion/s within the structure, the geometry of the system and the magnetic strength of the magnetic components, including that of the external magnetic field.

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(Some figures may appear in colour only in the online journal)

1. Introduction

Mechanical metamaterials are engineered systems that exhibit macroscopic properties (which are typically anomalous) that emerge due to the structure of their sub-units rather than their material composition [1, 2]. Examples of such metamaterials are systems which exhibit a negative Poisson's ratio, known as auxetics [3–8], which have the unusual property of expanding in the lateral direction when uniaxially stretched. Several models have been proposed to explain auxeticity, including the well known mechanism involving flexure or hinging of re-entrant honeycombs [9], or the relative rotation of rigid or semi-rigid units [5, 10–17], such as the rotating rigid squares model [5] and the rotating rigid triangles model [16, 17]. Whilst it is known that auxetic behaviour may contribute to an enhancement of the electromagnetic properties [18–20], work on auxetics systems which have magnetic insertions is still in its infancy [21–29].

Here we propose novel systems having magnetic insertion/s embedded within a non-magnetic matrix such that any re-orientation of the magnetic component/s (either by means of a mechanical force or by an externally applied magnetic field) results in a change in the geometry of the system giving rise to changes in the macroscopic properties including the Poisson's ratio and other mechanical properties which are sometimes anomalous. These systems can have various applications, including tunable filters which change their aperture according to the applied external magnetic field, in sensors which detect the amount of deformation occurring by means of an electromagnetic signal and in systems which require an instantaneous change in their mechanical properties such as the Poisson's ratio.

[4] www.um.edu.mt/science/metamaterials.

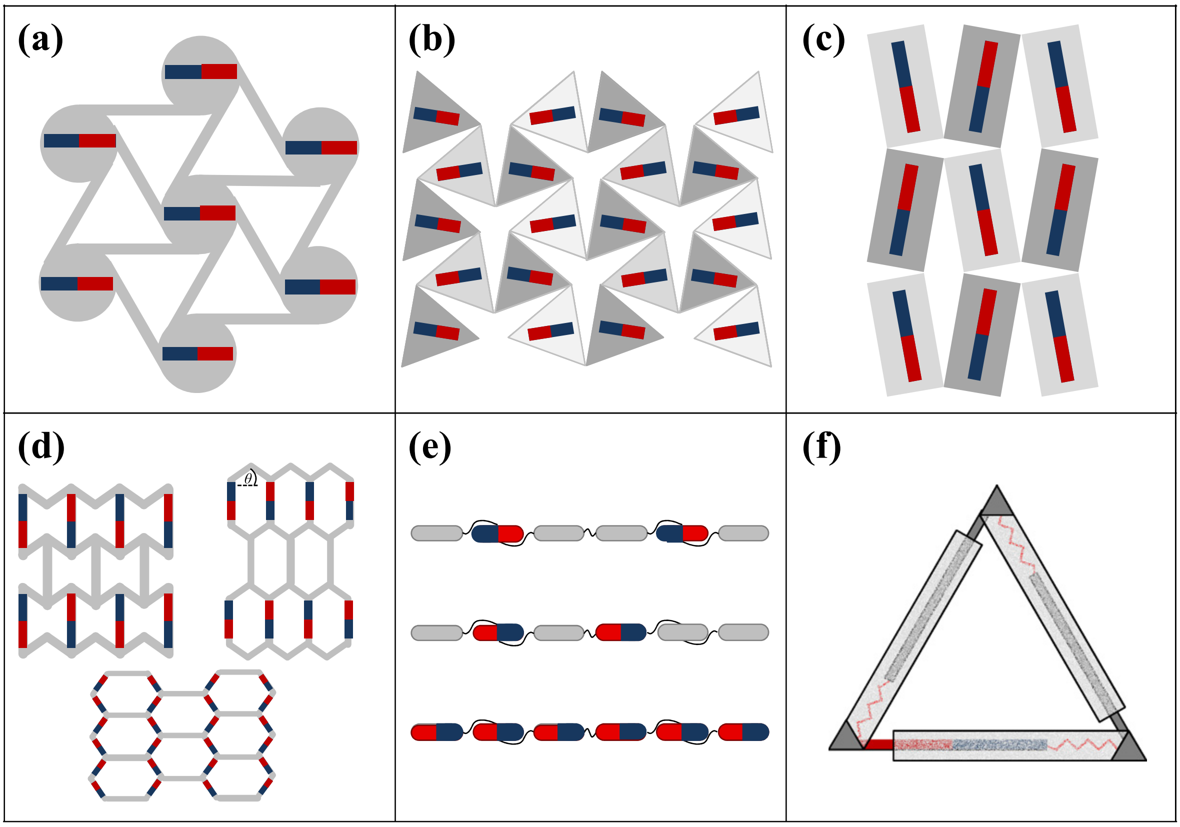

Figure 1. Various auxetic structures which lend themselves well to hosting magnetic insertion/s: (a) chirals, (b) rotating rigid triangles, (c) rotating rigid quadrilaterals, (d) honeycombs, (e) liquid crystalline polymers and (f) dilating triangles.

2. Concept

The concept proposed here is not dissimilar to what happens when one places a compass in a magnetic field where the needle rotates so as to align itself with the magnetic field. Here the compass needle is replaced by a magnet which is affixed to or embedded in a non-magnetic unit of the auxetic system. If the structure is adequately constrained so as not to translate or rotate as a rigid body, then on application of an external magnetic field the magnetic insertion re-orients itself, rotating alongside the structural unit, causing the whole structure to change its shape. In other words, the incorporated magnetic component when placed in an externally applied magnetic field may cause the system to deform and adopt a new equilibrium conformation and hence change its mechanical properties.

There are various auxetic structures, as illustrated in figure 1, which lend themselves well to hosting magnetic insertion/s so as to enable their geometry and properties to be controlled via internal magnetic insertion/s and/or externally applied magnetic fields. Such structures include chirals [30–32], honeycombs [9], STAR systems [33], liquid crystalline polymers [34], dilating systems [35], rotating rigid triangles [16, 17], Kagome lattices [27] and quadrilaterals [11–15]. For example, Wojciechowski [26] and Grima *et al* [28, 29] have recently proposed various systems incorporating magnetic components, such as the one in figure 2(a) (see also the animation in electronic format available at stacks.iop.org/SMS/22/084016/mmedia), which can respond, adapt and change their properties when exposed to externally applied electromagnetic fields [25, 26, 28, 29]. More recently, Ruzzene [27] has also proposed a Kagome lattice with cylindrical magnetic insertions which are perpendicular to the lattice plane where the lattice consists of a flexible Kagome lattice connected via thin and flexible connections; this system also exhibits a number of fascinating properties, including bi-stability and multi-stability. These various geometries, alongside their 3D counterparts, such as systems made from connected cuboids (see figure 2(b)) [36], can be such that the distribution and orientation of the magnetic insertion/s is arbitrarily providing additional tunability.

The magnetic insertion/s need not necessarily be of the permanent magnetic type but they can also be ferromagnetic, diamagnetic, paramagnetic or electromagnetic. If we take into consideration the latter case, the properties arising due to magnetism can be switched on or off and even controlled electrically making a system with versatile mechanical properties which can be used in applications requiring an instantaneous change or fine tuning in these mechanical properties. Such systems involving internal electromagnets can operate even in the absence of an external magnetic field, possibly even controlled remotely and resulting in a truly multi-functional tunable smart mechanical system. For

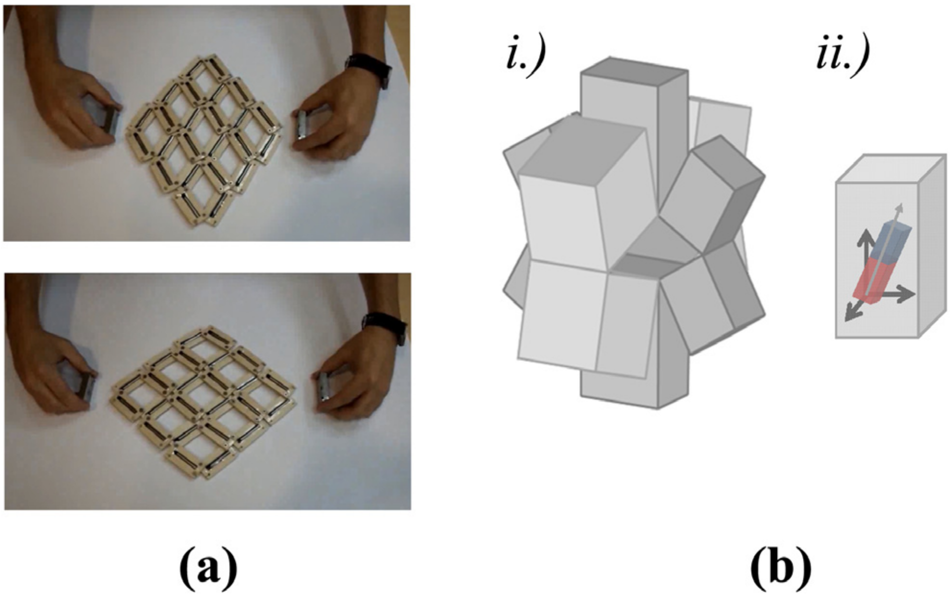

Figure 2. (a) Two screenshots from an experiment showing how systems with multiple magnetic insertions are affected on application of an external magnetic field (the full video can be found in the electronic supplementary information available at stacks.iop.org/SMS/22/084016/mmedia). The orientation of the internal magnets is as in figure 1(c). (b) (*i*) A 3D structure which can host magnetic insertions, where all or some of the units may have magnetic insertions placed in an arbitrary orientation as shown in (*ii*).

all of these systems, one can envisage even more versatile systems by changing various parameters within the systems. Additional versatility would obviously also be observed if the matrix itself, or parts of it, had some kind of magnetic property.

Before proceeding any further, it is important to point out that systems containing several magnetic insertions, apart from their interesting properties, will also behave differently from systems with just one magnetic insertion, even in the absence of an external magnetic field, since the multiple magnetic insertions may interact with each other resulting in a change of structure and hence of mechanical properties. This change in structure would be the result of the interplay between the stiffness of the hinges making up the mechanical system and the strength of the magnetic insertions. For example, let us consider rotating rigid quadrilaterals with a magnetic insertion in each sub-unit having the configuration as shown in figure 1(c). In the absence of an external magnetic field, if the magnetic insertions are strong enough, they will attract each other, causing the system to close its angle of aperture. Hence, if one tries to extend a closed configuration, more energy is required to extend the system since one has to overcome the attractive force of the magnets. Likewise, if one starts with an open configuration, less energy is required to contract the system since such a contraction will be aided by the attractive force of the internal magnets. With such magnetic insertions, one particular configuration is favoured.

Another feature observed in certain geometries containing multiple magnetic insertions is bi-stability or multi-stability [25–28]. Let us consider the former case, which can be observed in the honeycomb geometry (as shown in figure 1(d)), where the magnets attract each other along the horizontal direction, the attraction being strongest at angle θ close to $-90°$ (for re-entrant honeycombs) and angle θ close to $90°$ (for non re-entrant honeycombs). Hence, such magnetic honeycomb systems would have two points of stability. Other systems, such as rotating rigid triangles (as shown in figure 1(b)) [26] and flexible Kagome lattices [27] show the property of multi-stability as well as bi-stability.

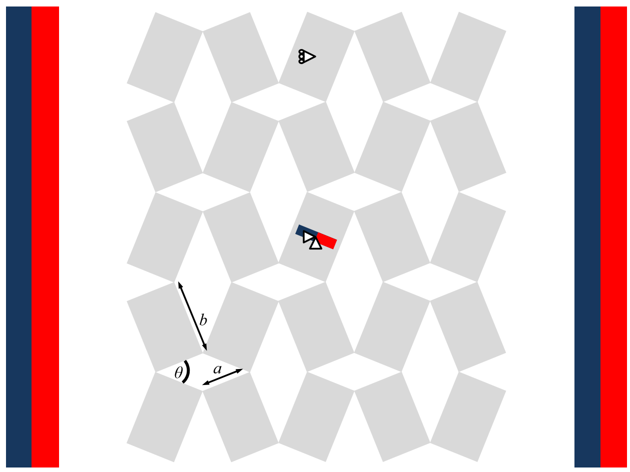

Figure 3. The 2D open boundary finite system that is taken into consideration in the case study.

Until now we have considered parameters affecting only the magnetic part of the system. However there are other parameters which govern the non-magnetic auxetic system. For example, if one considers the rotating rigid quadrilateral shown in figure 3, the non-magnetic matrix itself has a number of variables such as the lengths a and b, the interior angles of the quadrilateral (shown as a rectangle in figure 3), the connectivity, the angle of aperture θ, the hinging stiffness constant K_h and even the number of tessellated unit-cells, as discussed elsewhere [37]. These variables together with the magnetic variables will give rise to the final mechanical properties of the system. Also, magneto-auxetic systems could be engineered in a manner such that the magnetic insertion/s could influence one or more of the deformation mechanisms taking place, with the result that a new profile of mechanical properties would be achieved under different magnetic conditions. Here it should also be emphasized that given the large number of variables pertaining to the structural and magnetic components of the systems, any given model can be tailor-made to exhibit some specific set of properties. These properties will be dependent upon the various parameters making up the systems.

For an adequately constrained system having magnetic insertion/s, on applying an external magnetic field, the aperture of the system may change accordingly. This change in aperture, apart from resulting in a change in the dimensions of the system, may bring about a change in a number of other macroscopic properties which depend on the degree of aperture of the system, ranging from mechanical properties, such as the Poisson's ratios and the pore size, with obvious practical implications. For example, if this system were used as a filter, one could easily solve the problem of blocked pores by the temporary application of an externally applied magnetic field so as to de-foul the filter. Also, one could easily envisage filters built on these concepts where their pore sizes could be fine-tuned by the application of externally applied magnetic fields. Using the same principles, one could also foresee the use of such systems in the bio-medical industry. The system would be used as a medium for controlled drug release where the supporting matrix is opened by means of a magnetic field to release the active ingredients previously locked within it. This could have some very important applications in medicine, particularly in oncological treatments where, very often, the success or otherwise of the treatment depends on the ability of the effective targeted drug release so as to treat only cancerous cells rather than the surrounding healthy tissue. This concept can also be applied in the deployment of stents which can be 'inflated' (or 're-inflated') as needs be through the application of an external magnetic field.

Also since the deformation of the geometry of the system with magnetic insertions brings about a change in the magnetic flux, an induced electric current is generated around the system (from Lenz's law) and this induced current depends on the amount of deformation of the system. This property can be utilized in sensors which can detect the amount of deformation occurring within the system.

It is beyond the scope of this work to discuss the full range of tunable macroscopic properties afforded by these systems with magnetic insertions or to discuss the full range of systems which can lend themselves as base structures for achieving tunability of the macroscopic properties through the method discussed here. Instead, at this stage we shall just limit the discussion to one particular case, that of type I rotating rectangles with just one magnetic insertion, and discuss how this system can achieve tunable Poisson's ratios and pore sizes.

3. Case study—rotating rectangles with one magnetic insertion

In this study we shall consider a 2D open boundary finite system made up from $N_1 \times N_2$ perfectly rigid non-magnetic rectangles of dimensions $a \times b$ connected from their vertices in the type I conformation [12], as in figure 3. The rigid non-magnetic rectangle found in the centre of the system contains within it a rectangular magnet of dimensions $d_a \times d_b$ placed at its centre, as illustrated in figure 3, and is also pinned at its centre so that it cannot translate. In fact, the structure as a whole is aligned and pinned in 2D space, such that it cannot move (translate or rotate) as a rigid body. Such constraints can be imposed, for example, by pinning the structure as shown in figure 3, i.e. pinned at its centre so as to prevent translation together with another roller positioned in an adjacent tessellated rectangle just above it, which prohibits movement in the horizontal direction, thus effectively prohibiting rotation as a rigid body. Also, the rigid rectangles are assumed to have a unit thickness in the third direction and are connected together using simple two-dimensional hinges having stiffness constant K_h.

In an attempt to simulate the properties of this system, it is also assumed that the embedded magnet has a magnetic moment $\boldsymbol{\mu}_k$ $(k = 1, 2, \ldots, N)$ and is represented by a finite number $L = L_a \times L_b$ of single-point magnetic dipoles each having a magnetic moment $\boldsymbol{\mu}_{\text{int}}$ of magnitude:

$$|\boldsymbol{\mu}_{\text{int}}| = \frac{1}{L}|\boldsymbol{\mu}_k|. \quad (1)$$

In the simulation it is assumed that the point dipoles are separated from each other by a distance s_a along the a direction and s_b along the b direction in such a way that:

$$\frac{d_a}{L_a} = s_a \qquad \text{and} \qquad \frac{d_b}{L_b} = s_b \quad (2)$$

where L_a and L_b are the number of dipoles along the sides of the rectangles of dimensions a and b respectively. The distances, s_a and s_b, are chosen in such a manner that the net magnetic field arising from these point dipoles is not dissimilar from the magnetic field which would arise from a bar magnet of the same dimensions and would become more similar as L increases, i.e. by having a denser mesh of the single-point magnetic dipoles. Here it should be noted that since the magnetic insertion is embedded within the rectangle in a fixed manner, the orientation of the associated rectangle follows the magnetic moment $|\boldsymbol{\mu}_{\text{int}}|$ of the magnet if this rotates.

In addition to the internal magnet, it is also assumed that there is an external magnetic field which is generated by two additional large magnets, labelled as magnets X and Y respectively, placed on the sides of the system such that they are equidistant from the central rectangle, where each external magnet has a magnetic moment $\boldsymbol{\mu}_{\text{ext}}$ and is represented by L_{ext} dipoles. The separation between the external magnets is such that tips of the non-magnetic matrix will touch these magnets when the system is in its fully open configuration along the Ox_1 direction.

Noting that all the dipoles within the magnet found at the centre of the system are experiencing the magnetic field generated by the dipoles within the external magnets X and Y, the total magnetic potential energy of the system open at a certain angle θ is defined as the sum of the scalar products:

$$U_{\mathrm{Mag}} = -\sum_{i=1}^{L} (\boldsymbol{\mu}_{\mathrm{int}} \cdot \mathbf{B}_i) \tag{3}$$

where $\mathbf{B}_i$ represents the magnetic flux density (in tesla, commonly also known as the magnetic field) generated at location $\mathbf{r}_i$, the position vector of the dipole i within the central magnet, by all other magnetic dipoles from the two external magnets (adapted from [38]):

$$\mathbf{B}_i = \sum_{j=1}^{L_{\mathrm{ext}}} \frac{\mu_o}{4\pi}\left(\frac{3[\boldsymbol{\mu}_{\mathrm{ext}}\cdot(\mathbf{r}_{Xj}-\mathbf{r}_i)](\mathbf{r}_{Xj}-\mathbf{r}_i)}{|\mathbf{r}_{Xj}-\mathbf{r}_i|^5} - \frac{\boldsymbol{\mu}_{\mathrm{ext}}}{|\mathbf{r}_{Xj}-\mathbf{r}_i|^3}\right) + \sum_{j=1}^{L_{\mathrm{ext}}} \frac{\mu_o}{4\pi}\left(\frac{3[\boldsymbol{\mu}_{\mathrm{ext}}\cdot(\mathbf{r}_{Yj}-\mathbf{r}_i)](\mathbf{r}_{Yj}-\mathbf{r}_i)}{|\mathbf{r}_{Yj}-\mathbf{r}_i|^5} - \frac{\boldsymbol{\mu}_{\mathrm{ext}}}{|\mathbf{r}_{Yj}-\mathbf{r}_i|^3}\right). \tag{4}$$

where μ_0 is the permeability of free space.

It may be inferred from equation (3) above that for this given system, the total magnetic potential energy, U_{Mag} is dependent on the position and strength of the magnetic dipoles, their distance from each other depending on the angle θ between the rectangles. For example, in this particular case study, the magnetic energy U_{Mag} was evaluated for different values of $|\boldsymbol{\mu}_{\mathrm{ext}}|$ for the particular system shown in figure 3 where $N_1 \times N_2 = 5 \times 5$, $a \times b = 1.0$ cm $\times$ 1.5 cm, $d_a \times d_b = 0.8$ cm $\times 0.2$ cm, $l_a \times l_b = 4 \times 4$, $|\boldsymbol{\mu}_{\mathrm{int}}| = 0.15$ A m^2 (a typical value for a short bar magnet), $L_{\mathrm{ext}} = 66$. The values obtained for the magnetic potential energy U_{Mag} are plotted against the angle θ in figure 4.

In the absence of an external magnetic field, the magneto-auxetic system presented here will remain in the initial angle configuration unless it is made to change by some external force. The plots clearly show that on applying an external magnetic field (having the orientation as in figure 3), assuming that the hinges in the system are perfect pin-joints which offer no resistance, then the angle at which the magnetic energy potential is at a minimum corresponds to $\theta = 0°$. When the system is at $\theta = 0°$, the rectangular conformation is fully closed along the Ox_1 direction, meaning that the internal magnet would cause the central rectangle to rotate anti-clockwise. Due to the geometric constraints applied to the structure, on rotating, the central rectangle will force all the other rectangles to rotate with it so as to adopt the configuration with this lower U_{Mag}. Whether in reality this extreme value for the angle is achievable would, amongst other things, depend upon the stiffness of the hinges in the system.

In fact, in a real system, the stiffness of the hinges in the system offers resistance to rotation. In particular, it may be assumed that the stiffness in the hinges connecting the rectangles can be quantified in terms of the hinging constant K_{h} defined by:

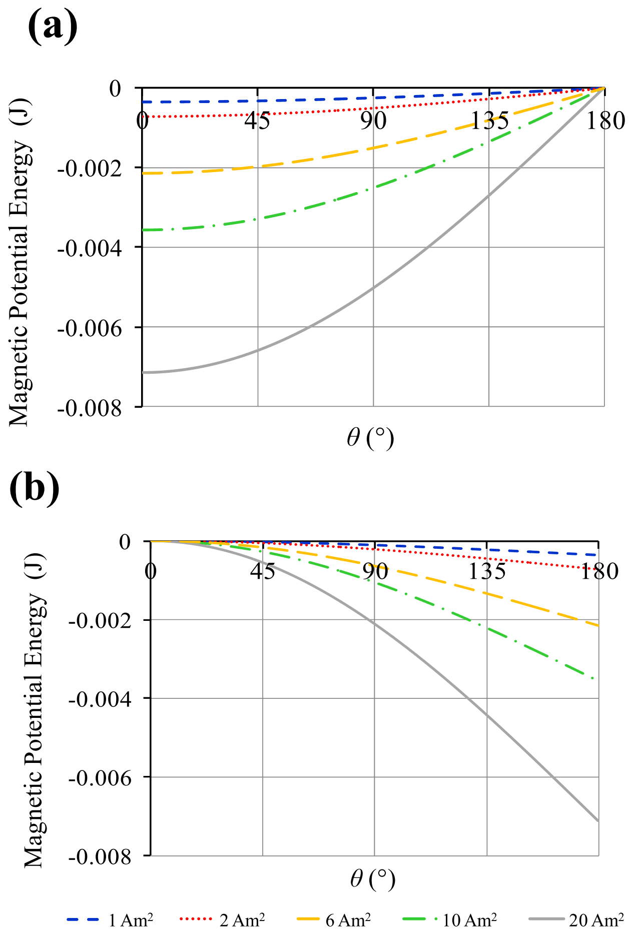

Figure 4. Magnetic potential energy U_{Mag} plotted against the angle of aperture θ for externally applied magnetic fields generated by magnets of varying $|\boldsymbol{\mu}_{\mathrm{ext}}|$: (a) for the configuration of the system as in figure 3, (b) for the system having an internal magnet with reversed poles.

$$U_{\mathrm{mech}} = \tfrac{1}{2} K_{\mathrm{h}} (\theta_{\mathrm{fin}} - \theta_{\mathrm{init}})^2 \tag{5}$$

where U_{mech} is the energy required to bring about a change from θ_{init} to θ_{fin}.

Thus, given a system which, in the absence of any magnetic insertion or other externally applied forces would be in its 'relaxed state' when the rectangles are at an angle θ_{init}, then with the insertion of a magnet and the application of an external magnetic field, the rectangles would re-orient themselves to a configuration having the rectangles at an angle θ_{fin} where the value of θ_{fin} corresponds to the angle where there is an equilibrium between the magnetic potential energy and the mechanical energy. In other words, through careful control of the stiffness constant and the strength of the magnetic components, one may fine tune the extent to which the change in angle occurs.

In particular, assuming that:

(i) the change in the magnetic contribution to the internal energy of the system is independent of the path along which this change is occurring and thus the work done on the system by the magnets may be estimated by:

$$W_{Mag} = \left[U_{Mag}(\theta = \theta_{fin}) - U_{Mag}(\theta = \theta_{init})\right]; \quad (6)$$

(ii) the work done by the system against the stiffness of the hinges, recognizing that there are $N_h = 2N_1N_2 - N_1 - N_2$ hinges, may be estimated by:

$$W_{Mech} = N_h \tfrac{1}{2} K_h (\theta_{fin} - \theta_{init})^2 \quad (7)$$

then θ_{fin}, the angle θ at which the new energy equilibrium occurs due to the presence of the magnetic components but no other additional external forces, may be estimated by using the expression:

$$W_{Total} = W_{Mag} + W_{Mech}. \quad (8)$$

Using equation (8), the equilibrium angle θ_{fin} for the system is found when $W_{Total} = 0$. (The value of W_{Total} indicates whether work is being done by the system, i.e. when $W_{Total} < 0$, or on the system, when $W_{Total} > 0$. Systems with $W_{Total} = 0$ indicate a state of equilibrium. Note that some systems may achieve a locked conformation when $\theta = 180°$ or 0° prior to reaching a state of $W_{Total} = 0$.) This change in equilibrium angle depends on the interplay of three factors, the strength of the magnetic insertion, the strength of the external magnetic field applied and the stiffness of the hinges in the structure. Depending on the values of these three factors, the angle at which the system is in equilibrium may vary.

For the particular system discussed here, as illustrated in figure 5(a), there is always a tendency for the rotations to occur in such a way that the equilibrium angle θ decreases on increasing the strength of the magnetic field. This occurs due to the orientation of the internal magnet with respect to the external magnetic field (as shown in figure 3) where the internal magnet will try to re-orient itself by rotating anti-clockwise, turning with it the rectangle within which it is embedded. Due to geometric constraints of the structure this rotation forces all the other rectangles to rotate in a manner which decreases the angle θ between the rectangles. In fact, had the rectangles been connected through hinges which offer no resistance ($K_h = 0$), then the structure would have closed to $\theta = 0°$.

Obviously this behaviour where the internal magnet rotates anti-clockwise, hence forcing the structure to close, only occurs if the internal magnet is oriented with respect to the external magnetic field as in figure 3. In fact, if the poles of the internal magnet had to be reversed, then there would have been the tendency for this angle θ to increase (as shown in figure 5(b)). Also, different relationships would have been obtained if the external magnetic field had to be aligned in different directions relative to the system.

All this is very significant since the mechanical properties, such as the Young's modulus and the Poisson's ratio, are dependent on the degree of aperture of the presented system. In the case of the Poisson's ratio, Grima *et al* [12] have shown that for connected rectangles of dimensions $a \times b$ connected together as in figure 3, the Poisson's ratio is dependent on the degree of aperture, measured through the angle θ between the rectangles, and is given by:

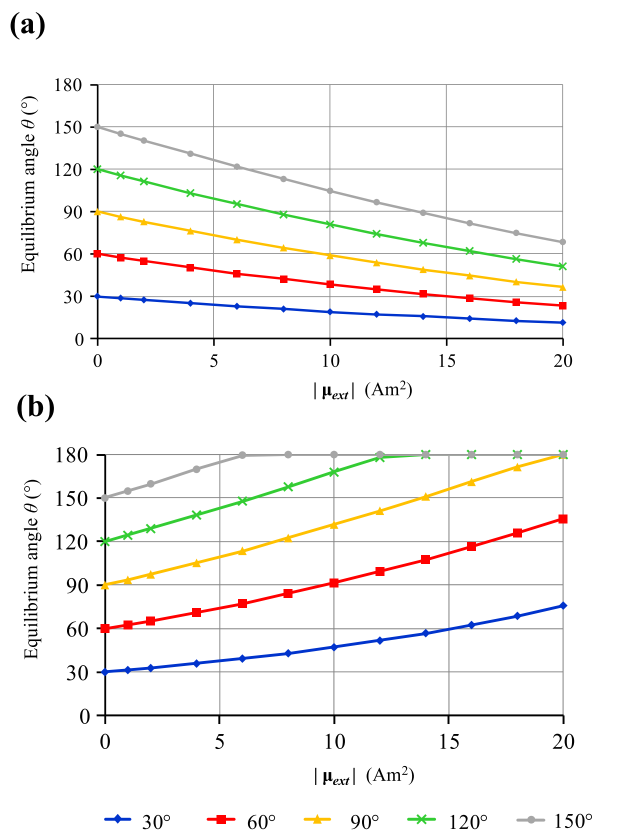

Figure 5. Equilibrium angle between the magnetic and mechanical system on varying the externally applied magnetic field generated by magnets of varying $|\boldsymbol{\mu}_{ext}|$ for systems of different θ_{init}: (a) for the system as depicted in figure 3, (b) for the system having an internal magnet with reversed magnetic poles.

$$\nu_{12} = (\nu_{21})^{-1} = \frac{a^2\cos^2\left(\frac{\theta}{2}\right) - b^2\sin^2\left(\frac{\theta}{2}\right)}{a^2\sin^2\left(\frac{\theta}{2}\right) - b^2\cos^2\left(\frac{\theta}{2}\right)}. \quad (9)$$

This equation clearly suggests that for a given system, the magnitude and sign of the Poisson's ratio depends on the angle between the rectangles. Hence, if on applying an external magnetic field one changes the angle between the rectangles, then the Poisson's ratio of such a system can be controlled through an external magnetic field, with the many obvious practical advantages that such control brings with it.

In fact, as shown in figure 6, the Poisson's ratio for the particular system as described in figure 3 having a hinging constant K_h of 1×10^{-4} J rad^{-2} changes significantly on varying the magnitude of the external magnetic field applied. For example, taking into consideration the system with $a \times b = 1.0$ cm $\times$ 1.5 cm when in the absence of an external magnetic field it has an angle of 90° and a Poisson's ratio $\nu_{12} = +1$, on applying an external magnetic field and increasing the magnitude of such a field, the Poisson's ratio becomes negative and reaches a value of -0.35 when the external magnetic field is generated by external magnets having a dipole moment $|\boldsymbol{\mu}_{ext}|$ of 20 Am^2. On the other hand, for a system which in the absence of an external magnetic field

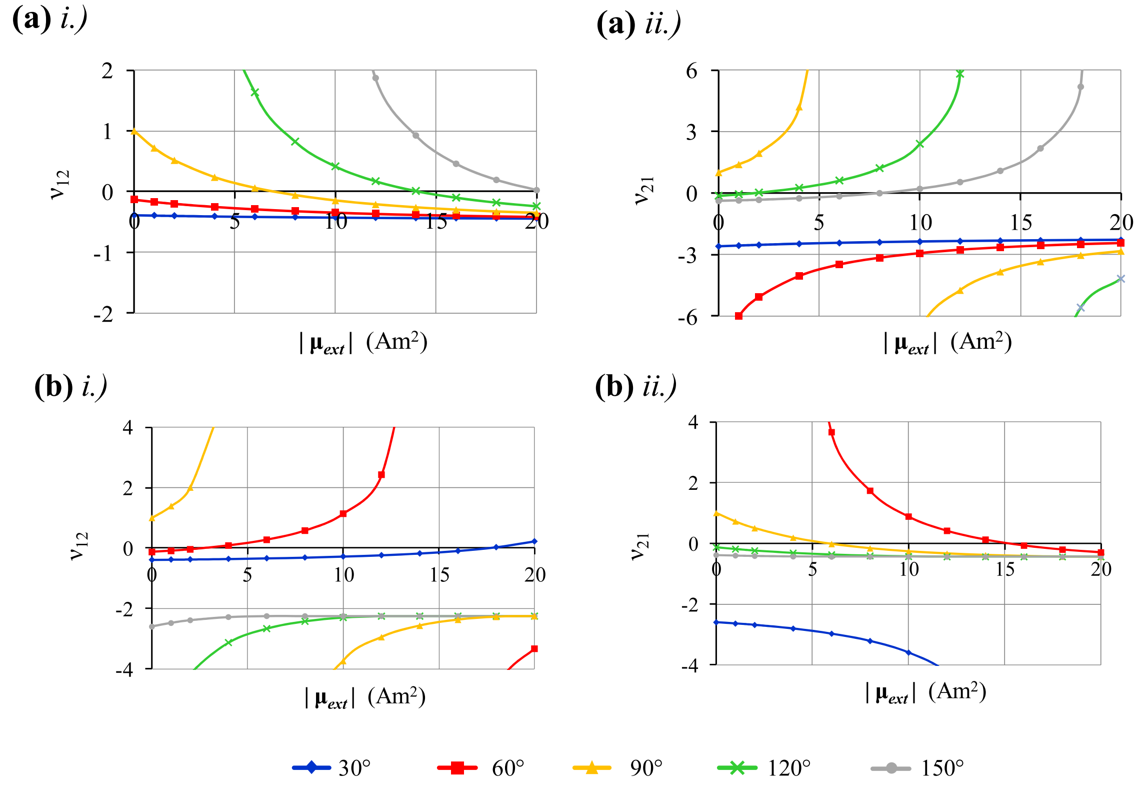

Figure 6. The Poisson's ratios (*i*) ν_{12} (*ii*) ν_{21} on varying the externally applied magnetic field generated by magnets of varying $|\boldsymbol{\mu}_{\text{ext}}|$ for systems of different θ_{init}: (a) for the system as depicted in figure 3, (b) for the system having an internal magnet with reversed magnetic poles.

has an angle of 150° and a Poisson's ratio $\nu_{12} = -2.60$, on applying an external magnetic field generated by external magnets having a dipole moment $|\boldsymbol{\mu}_{\text{ext}}|$ of 12 Am^2, the Poisson's ratio of the system has a value of +1.87. From these plots it is observed that on application of an external magnetic field, certain values of Poisson's ratios are favoured, meaning that with a magnetic insertion, one can fine tune the system to adopt certain ranges of Poisson's ratios but not others. Obviously it is not just the type I rotating rectangles scheme which lends itself to a tunable Poisson's ratio through the mechanism discussed here. In fact any other system which can change its Poisson's ratio as a result of a change in its geometry can also exhibit some form of tunability. Also, systems could be engineered in such a manner that the magnetic insertion/s could influence one or more of the deformation mechanisms taking place with the result that a new profile of mechanical properties would be achieved under different magnetic field conditions.

At this point it should be highlighted that it is not only the Poisson's ratio which is affected by the application of an external magnetic field. In fact, as discussed elsewhere [39] there are also significant changes in the pore dimensions as the angle of aperture changes, something which in this case may be brought about by applying or changing the magnitude of the external magnetic field. This makes the system presented here an ideal candidate for smart filters which, for example, can be very easily de-fouled by means of an external magnetic field or could change their pore size.

More specifically, the system made from $a \times b$ sized rectangles has two sets of pores in the shape of a rhombus with a side length that is equal to that of the rigid rectangles and internal angles equal to the angles between the rectangles. For any porous system, one can define the pore size in terms of ρ, the radius of the largest spherical particles that can pass through, which for the system presented here having two different sets of pores, one with dimensions a, a and one with dimensions b, b, is given by:

$$\rho = \frac{\max\{a, b\}}{2} \sin(\theta) \qquad (10)$$

where for this particular system, θ can be varied by changing the magnitude of the external magnetic field. It may be inferred from equation (10) that the largest pore size occurs when the system has an angle of aperture of 90°. This relationship between ρ and the externally applied magnetic field is clearly illustrated in figure 7, which confirms that the pore size of the system, (ρ), is affected by the magnitude of the applied magnetic field where, for example, taking into consideration the system which in the absence of a magnetic field has an angle of 150°, ρ has a value of 0.375, but on applying an external magnetic field generated by external magnets having a dipole moment $|\boldsymbol{\mu}_{\text{ext}}|$ of 14 Am^2 this value increases all the way to 0.75 until it starts to decrease again. On the other hand, for the system which in the absence of a magnetic field has an angle of 90°, ρ is at the maximum value of 0.75, but on applying an external magnetic field generated by external magnets having a dipole moment $|\boldsymbol{\mu}_{\text{ext}}|$ of 20 Am^2 this value decreases to 0.45.

Here it should be mentioned that deformations in the structure that are brought about through the application of a

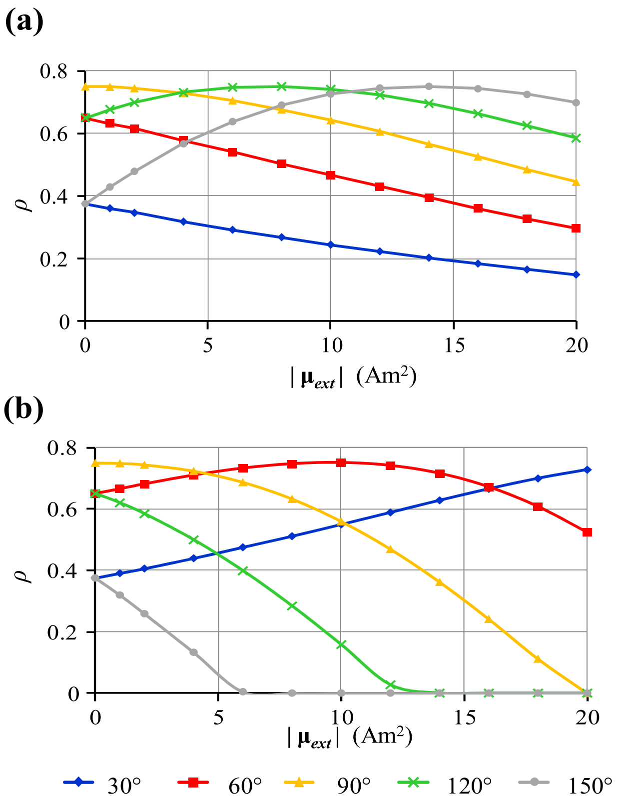

Figure 7. The radius ρ of the largest spherical particles that can pass through the pores of the system on varying the externally applied magnetic field generated by magnets of varying $|\boldsymbol{\mu}_{\text{ext}}|$ for systems of different θ_{init}: (a) for the system as depicted in figure 3, (b) for the system having an internal magnet with reversed magnetic poles.

magnetic field can permit changes in pore size of the structure which could not have been achieved by just mechanical pulling as a result of the locking that would occur in such cases. In fact, as discussed elsewhere [12], in the absence of internal and external magnets, on deforming the system by application of a uniaxial force along the Ox_i direction the system will lock at a certain angle θ_i^* where:

$$\text{deforming along the } Ox_1 \text{ direction } \theta_1^* = 2\tan^{-1}\left(\frac{b}{a}\right) \quad (11)$$

$$\text{deforming along the } Ox_2 \text{ direction } \theta_2^* = 2\tan^{-1}\left(\frac{a}{b}\right). \quad (12)$$

Hence for a rectangular system which is being extended by a uniaxial force along the Ox_i direction, given an initial configuration where $\theta = \theta_o < \theta_i^*$ the system will not be able to go above the angle of θ_i^* (due to locking) which means that the system cannot assume a fully open conformation of $\theta = 180°$ if $\theta_o < \theta_i^*$. Likewise if one is contracting a rectangular system by means of a uniaxial force along the Ox_i direction, given an initial configuration where $\theta = \theta_o > \theta_i^*$ the system will not be able to go below the angle of θ_i^* hence the system cannot assume a fully closed conformation of $\theta = 0°$ if $\theta_o > \theta_i^*$.

However, this is not the case for the magneto-mechanical system presented in this case study. On placing a single magnetic insertion at the centre rectangle of the constrained system and applying an external magnetic field, one is able to overcome the locking angle, effectively making the system free to deform along the whole range of angles without any hindrance. This occurs since the internal magnetic insertion placed at the centre rectangle will cause the system to rotate to a new equilibrium angle and the rotation will not be affected by the locking angle mentioned above. This feature further enhances the performance of such systems if used as tunable filters.

Obviously the properties presented in figure 5–7 only relate to a 5 × 5 system having the dimensions as specified above. If one had to keep the same dimensions of the magnets, rectangles etc and only vary the number of rectangular units used whilst keeping the external magnets at a location such that the tips of the non-magnetic matrix will touch these magnets when the system is in its fully open configuration along the Ox_1 direction, then the properties afforded by the system would also vary (as shown in figure 8). This is due to the fact that in such systems, the number of hinges offering resistance to rotation would have changed as would have the distance between the central magnet and the two external magnets. It is also envisaged that additional variation could be introduced if one uses different values for the other variables in the system or combinations thereof. Such systems, as well as related ones, could still be simulated using the simulation method developed here, since the methodology used is a general one which permits full control of all the parameters. Thus, this method used here to simulate the properties of this particular magneto-mechanical metamaterial can also be applied and adapted to simulate the other systems proposed.

Before we conclude, we must mention the fact that the behaviour discussed here is only applicable to a system having just one magnetic insertion positioned as discussed above. More complex and even more interesting behaviour will arise if the system has more than one magnetic insertion since the internal magnets would affect each other even in the absence of an external magnetic field, as discussed elsewhere [25–29]. In fact, it should be once again highlighted that one can envisage more versatile systems by changing various parameters within the systems. For instance, one may vary the number of structural sub-units which contain a magnetic insertion, as well as the size and shape of each magnetic insertion/s, the magnetic strength of each magnetic insertion/s, and the position and 3D orientation in space of each magnetic insertion/s relative to the local 3D co-ordinate system of each non-magnetic structural sub-unit. In particular, it is possible to have magnets which are out-of-plane of the non-magnetic matrix and whose centre is not co-planar to the non-magnetic matrix; such systems would exhibit their own particular interesting characteristics. Moreover, one may also use super-lattices so as to permit more variation and vary the global 3D orientation of the structure relative to the external magnetic field. In the case of multiple magnetic insertions, one can further consider two cases: (*i*) where the internal magnetic insertions are far apart from each other such that they do not interact with each other, however these insertions are affected by the application of an external magnetic field; (*ii*) where the internal magnetic insertions are close enough to each other that they can interact with each other, i.e. apart

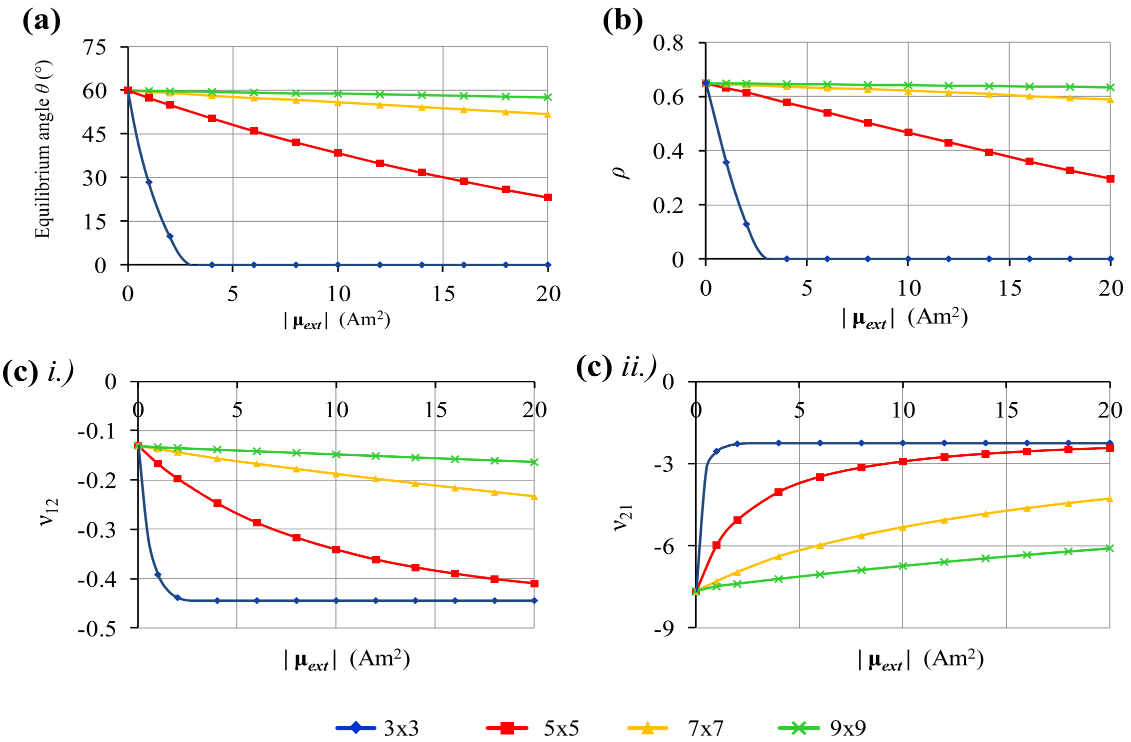

Figure 8. (a) The equilibrium angle, (b) pore size ρ and (c) the Poisson's ratios (*i*) ν_{12}, (*ii*) ν_{21} on varying the externally applied magnetic field generated by magnets of varying $|\boldsymbol{\mu}_{\text{ext}}|$ for the system as depicted in figure 3 having an initial angle of 60° but different numbers of rectangular units. Note that a 1 × 1 system would effectively work as a compass.

from being affected by an externally applied magnetic field, the magnetic insertions within the system will also affect the properties of the system in the absence of an external magnetic field. For example, for the rotating rigid quadrilaterals system with a magnetic insertion in each sub-unit (figure 1(c)), in the absence of an external magnetic field, if the magnetic insertions are strong enough, they will attract each other causing the system to change its angle of aperture. As a corollary of this, if the internal magnetic components had to be electromagnets, one may achieve the control discussed here just by varying the electrical current being supplied, with the obvious superior and more practical characteristics associated with this self-contained tunable system. Such systems would still benefit from tunable Poisson's ratios but could also have even more unique properties.

Also, it is not only auxetic systems that could benefit through the concepts proposed here since even non-auxetic systems, including ones which exhibit other forms of anomalous behaviour, could be similarly controlled.

4. Conclusion

This paper has shown that one can use magnetic components embedded in or affixed to the matrix of existing auxetic and related systems in order to achieve controlled modifications of the internal structure of these systems through the application of, for example, an external magnetic field. This could bring about a change in the macroscopic properties, including the Poisson's ratio, which would be a highly desirable property. We have shown that such properties can be fine-tuned by careful choice of the strength of the magnetic components used, including the external magnetic field applied, as well as the properties characteristic of the system, such as the mechanical stiffness constants. In this way the desired set of mechanical and other macroscopic properties can be tailor-made for particular practical applications. In view of the many potential applications that such smart systems may have [25–29], we hope that this work will provide an impetus to experimentalists who can manufacture materials and products based on the concepts proposed here. Such products could well result in a new generation of smart auxetic materials with tunable macroscopic characteristics.

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References

[1] Nicolaou Z G and Motter A E 2012 Mechanical metamaterials with negative compressibility transitions *Nature Mater.* **11** 608–13

[2] Grima J N and Caruana-Gauci R 2012 Mechanical metamaterials: materials that push back *Nature Mater.* **11** 565–6

[3] Lakes R S 1987 Foam structures with a negative Poisson's ratio *Science* **235** 1038–40
[4] Evans K E, Nkansah M A, Hutchinson I and Rogers S 1991 Molecular network design *Nature* **353** 124
[5] Grima J N and Evans K E 2000 Auxetic behavior from rotating squares *J. Mater. Sci. Lett.* **19** 1563–5
[6] Wojciechowski K W, Alderson A, Brańka A C and Alderson K L 2005 *Phys. Status Solidi* b **242** 497 (Preface: Phys. Status Solidi b 242/3)
[7] Alderson K L, Alderson A, Smart G, Simkins V R and Davies P J 2002 Auxetic polypropylene fibres: part 1—manufacture and characterisation *Plast. Rubber Compos.* **31** 344–9
[8] Alderson A 1999 A triumph of lateral thought *Chem. Indust.* **10** 384–91
[9] Gibson L J, Ashby M F, Schajer G S and Robertson C I 1982 The mechanics of two-dimensional cellular materials *Proc. R. Soc.* A **382** 25–42
[10] Sigmund O 1995 Tailoring materials with prescribed elastic properties *Mech. Mater.* **20** 351–68
[11] Grima J N, Alderson A and Evans K E 2004 Negative Poisson's ratio from rotating rectangles *Comput. Methods Sci. Technol.* **10** 137–45
[12] Grima J N, Gatt R, Alderson A and Evans K E 2005 On the auxetic properties of "rotating rectangles" with different connectivity *J. Phys. Soc. Japan* **74** 2866–7
[13] Attard D and Grima J N 2008 Auxetic behaviour from rotating rhombi *Phys. Status Solidi* b **245** 2395–404
[14] Attard D, Manicaro E and Grima J N 2009 On rotating rigid parallelograms and their potential for exhibiting auxetic behaviour *Phys. Status Solidi* b **246** 2033–44
[15] Grima J N, Manicaro E and Attard D 2011 Auxetic behaviour from connected different-sized squares and rectangles *Proc. R. Soc.* A **467** 439–58
[16] Grima J N and Evans K E 2006 Auxetic behavior from rotating triangles *J. Mater. Sci.* **41** 3193–6
[17] Grima J N, Chetcuti E, Manicaro E, Attard D, Camilleri M, Gatt R and Evans K E 2011 On the auxetic properties of generic rotating rigid triangles *Proc. R. Soc.* A **468** 810–30
[18] Smith F C, Scarpa F and Chambers B 2000 The electromagnetic properties of re-entrant dielectric honeycombs *IEEE Microw. Guid. Wave Lett.* **10** 451–3
[19] Lira C, Scarpa F, Olszewska M and Celuch M 2009 The SILICOMB cellular structure: mechanical and dielectric properties *Phys. Status Solidi* b **246** 2055–62
[20] Ciobanu R, Damian R and Casian-Botez I 2010 Electromagnetic characterization of chiral auxetic metamaterials for EMC applications *Comput. Stand. Interfaces* **32** 101–9
[21] Scarpa F, Bullough W A and Lumley P 2004 Trends in acoustic properties of iron particle seeded auxetic polyurethane foam *Proc. Inst. Mech. Eng.* C **218** 241–4
[22] Scarpa F and Smith F C 2004 Passive and MR fluid-coated auxetic PU foam—mechanical, acoustic, and electromagnetic properties *J. Intell. Mater. Syst. Struct.* **15** 973–9
[23] Dudek M R, Grabiec B and Wojciechowski K W 2007 Molecular dynamics simulations of auxetic ferrogel *Rev. Adv. Mater. Sci.* **14** 167–73
[24] Dudek M R and Wojciechowski K W 2008 Magnetic films of negative Poisson's ratio in rotating magnetic fields *J. Non-Cryst. Solids* **354** 4304–8
[25] Grima J N 2012 Auxetics and other mechanical metamaterials *Public Lecture Presented at the University of Zielona Góra (May 2012)*
[26] Wojciechowski K W 2012 Computer modelling of simple systems of counterintuitive mechanical properties *4th Int. Conf. and 9th Int. Workshop on Auxetics and Related Systems (Bolton, Sept. 2012)* p 24
[27] Ruzzene M 2012 Switchable auxetic topologies in flexible Kagome lattices *4th Int. Conf. and 9th Int. Workshop on Auxetics and Related Systems (Bolton, Sept.)* pp 25–6
[28] Caruana-Gauci R, Dudek M R, Wojciechowski K W and Grima J N 2012 Magnetic mechanical metamaterials *4th Int. Conf. and 9th Int. Workshop on Auxetics and Related Systems (Bolton, Sept. 2012)* p 22
[29] Caruana-Gauci R, Dudek M R and Grima J N 2012 On the properties of rotating rigid magnetic units *4th Int. Conf. and 9th Int. Workshop on Auxetics and Related Systems (Bolton, Sept. 2012)* p 55
[30] Prall D and Lakes R S 1997 Properties of a chiral honeycomb with a Poisson's ratio of −1 *Int. J. Mech. Sci.* **39** 305–14
[31] Grima J N, Gatt R and Farrugia P-S 2008 On the properties of auxetic meta-tetrachiral structures *Phys. Status Solidi* b **245** 511–20
[32] Cicala G, Recca G, Oliveri L, Perikleous Y, Scarpa F, Lira C, Lorato A, Grube D J and Ziegmann G 2012 Hexachiral truss-core with twisted hemp yarns: out-of-plane shear properties *Compos. Struct.* **94** 3556–62
[33] Grima J N, Gatt R, Alderson A and Evans K E 2005 On the potential of connected stars as auxetic systems *Mol. Simul.* **31** 925–35
[34] He C, Liu P and Griffin A C 1998 Toward negative Poisson ratio polymers through molecular design *Macromolecules* **31** 3145–7
[35] Rothenburg L, Berlin A and Bathurst R J 1991 Microstructure of isotropic materials with negative Poisson's ratio *Nature* **354** 470–2
[36] Attard D and Grima J N 2012 A three-dimensional rotating rigid units network exhibiting negative Poisson's ratios *Phys. Status Solidi* b **249** 1330–8
[37] Gatt R 2010 New structures and materials exhibiting negative Poisson's ratios and negative compressibility *PhD Thesis* University of Malta
[38] Coey J M D 2010 *Magnetism and Magnetic Materials* (Cambridge: Cambridge University Press)
[39] Attard D 2011 Investigations on materials and structures exhibiting negative mechanical properties *PhD Thesis* University of Malta