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# Connected Triangles Exhibiting Negative Poisson's Ratios and Negative Thermal Expansion

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Materials normally get thinner when stretched (positive Poisson's ratio) and expand when heated (positive thermal expansion coefficients). However, not all systems behave in this way and systems (materials or structures) which defy common expectation and become wider when stretched (negative Poisson's ratios, NPR, commonly referred to as auxetic[1–3]) or contract when heated[4,5] (negative thermal expansion, NTE) do exist and are now well documented.[1–5] Materials exhibiting one of these unusual properties are known to be very useful in various practical applications.[3,4,6,7] For example it has been shown that the presence of a negative Poisson's ratio gives auxetic materials various enhanced physical characteristics over their conventional counterparts ranging from increased indentation resistance and a natural ability to form dome-shaped surfaces[6] to improved acoustic damping properties.[7] Similarly, negative thermal expansion materials can be extremely useful in various applications, such as in the manufacture of composites exhibiting some pre-determined thermal expansion coefficients since the presence of a component with negative thermal expansion lowers the overall thermal expansion of a composite.[4]

Although in recent years considerable progress and developments were made in the distinct fields of NPR and NTE, so far, research on systems which could exhibit both of these anomalous properties simultaneously is still in its infancy. Nevertheless, it is known that NTE and NPR are not mutually exclusive properties and in fact, single-crystalline polyacetylene networks (hypothetical carbon allotropes, which so far have not been synthesised) have been predicted to exhibit both properties.[8]

Here we will show how existing knowledge on systems exhibiting negative Poisson's ratios and negative thermal expansion can be combined to produce a novel and easily constructible system which can exhibit both negative properties simultaneously.

In particular, we note that as we have recently shown, the construct made from beams connected as illustrated in Fig. 1(a) can exhibit NPR since when it is uniaxially stretched, the beams will flex as illustrated in Fig. 1(b) with the net effect that the triangles rotate relative to each other to produce a more open structure[9,10] (the "rotating triangles"

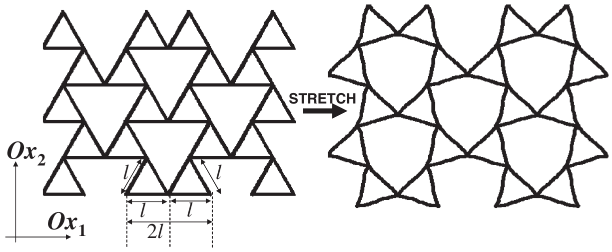


Fig. 1. The (a) un-stretched and (b) stretched form of the "flexing triangles" system which exhibits negative in-plane Poisson's ratio of −1 if the triangles are equilateral.

effect, resulting in negative Poisson's ratios of −1 in-plane if the triangles are equilateral.[9–11]) In such systems, the straight beams of length $2l$ can either be welded together or connected through the use of "pin joint"-type connections at ends and centres of the beams (corresponding to the vertices of the triangles) where the axes of the "pin joints" are orthogonal to the plane of the structure. The use of such "pin joints" is possible since the formation of the triangles reduces the degrees of freedom of the system and give the structure its characteristic shear rigidity. The presence of these triangles also provide us with a route for turning this auxetic structure into one which can also exhibit negative thermal expansion since as recently proposed,[12–14]) equilateral triangles constructed in such a way that their base is made from a material with a thermal expansion coefficient being at least four times that of the other two sides will get shorter when heated (the "shortening of triangles" mechanism). In fact, it has been shown that d$h$, the change in height $h$ when the equilateral triangle in Fig. 2 is subjected to a temperature change d$T$ is given by:[14])

$$dh = \frac{4\alpha_{S2} - \alpha_{S1}}{3} h \, dT$$

where $\alpha_{S1}$ is the thermal expansion coefficient of the material from which the base of the triangle is made and $\alpha_{S2}$ is the thermal expansion of the material from which the other two sides of the triangle are made. This "shortening of triangles" mechanism may be directly implemented into our system illustrated in Fig. 1(a) (with the different beams of length $2l$ connected together using pin-joints as described above) by, for example, making the beams in the $Ox_1$ direction having a different coefficient of thermal expansion

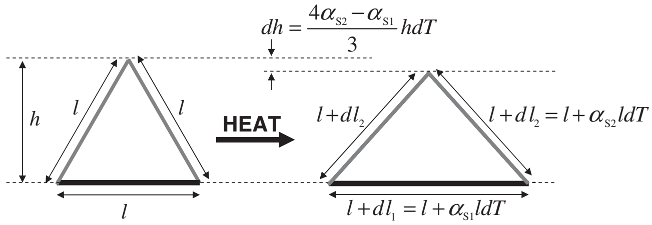


Fig. 2. The "triangles shortening" mechanism: Illustrated here is an equilateral triangular system[12–14]) where the base is made from material with thermal expansion coefficient $\alpha_{S1}$ and the other two sides are made from a material with thermal expansion coefficient $\alpha_{S2}$. This triangles becomes shorter in height $h$ when heated if $\alpha_{S1} > 4\alpha_{S2}$. Note that the sides of the triangles are connected together through pin joints.

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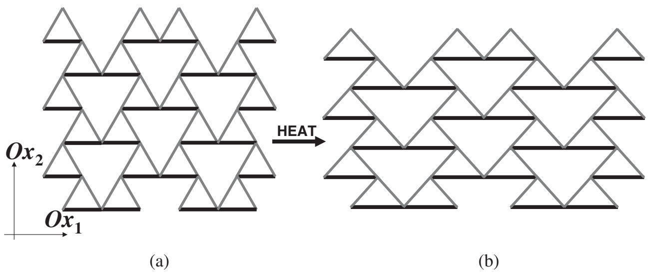


Fig. 3. An illustration of how the concept illustrated in Fig. 2 can be employed in our auxetic system (illustrated in Fig. 1) so that it can exhibit negative thermal expansion. Note that the beams parallel to the $Ox_1$ direction have a higher thermal expansion than the other beams (in this case, 8 times higher).

from the other beams as illustrated in Fig. 3. This slight variation in the manner of construction of this auxetic system will have a very significant effect on its thermal expansion properties which become anisotropic and possibly negative (i.e., thermal contraction, see Fig. 3). In fact, for the system with equilateral triangles illustrated in Fig. 3(a), if the coefficient of thermal expansion of the beams parallel to the $Ox_1$ is $\alpha_{S1}$ and the coeffcient of thermal expansion of the other beams is $\alpha_{S2}$, then the deformation of the structure as a result of a change in temperature d$T$ is defined by:

$$\begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{pmatrix} = \begin{pmatrix} \alpha_{S1} & 0 \\ 0 & \dfrac{4\alpha_{S2} - \alpha_{S1}}{3} \end{pmatrix} \mathrm{d}T$$

where $\varepsilon_{11}$ is the strain in the $Ox_1$ direction, $\varepsilon_{22}$ is the strain in the $Ox_2$ direction and $\varepsilon_{12} = \varepsilon_{21}$ is equal to half the shear strain. Thus using standard axis transformation techniques,[15] the coefficient of thermal expansion of the structure at an angle $\zeta$ to the $Ox_1$ axis is given by:

$$\alpha(\zeta) = \alpha_{S1} \cos^2 \zeta + \frac{4\alpha_{S2} - \alpha_{S1}}{3} \sin^2 \zeta$$

These equations clearly suggests that the system is anisotropic with respect to its thermal expansion and that when $\alpha_{S1} > 4\alpha_{S2}$, the thermal expansion coefficient of the system can assume negative values with a minimum at $\zeta = 90°$ (i.e., the $Ox_2$ direction) thus confirming that this auxetic structure (the structure is still capable of exhibiting negative Poisson's ratio through the "rotating triangles" mechanism involving flexure of the beams) can also exhibit NTE. Furthermore, when $\alpha_{S2} > 4\alpha_{S1}$, the maximum thermal expansion coefficient of the system (which also occurs when $\zeta = 90°$) is greater than any of the individual $\alpha_S$'s, another useful thermal property in its own accord.

It should also be noted that it is possible to fine-tune the thermal and mechanical properties of such a system to predetermined values by, for example, changing the relative lengths of the sides of the triangles which would affect the magnitude of both the Poisson's ratio[10] and the thermal expansion coefficient. The thermal properties can also be further adjusted if all the three sides of the triangles are made from materials having different thermal expansion coefficients.

To conclude, we have shown that the system in Fig. 3 is capable of exhibiting both negative Poisson's ratio (through the "rotating triangles" mechanism involving flexure of the beams) and negative thermal expansion (through the "shortening of triangles" mechanism), where the exact magnitudes of these properties can be altered through construction. In view of the many beneficial effects associated with having either a negative Poisson's ratio or a negative thermal expansion coefficient, we envisage that our newly proposed model system which exhibits both these properties simultaneously may find uses in various practical applications. We also hope that this report will stimulate more research in the fields of auxetics and negative thermal expansion so that more systems which can exhibit both properties simultaneously will be identified.

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