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AUXETIC CELLULAR MATERIALS AND STRUCTURES

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ABSTRACT

Auxetic materials and structures exhibit the very unusual property of becoming wider when stretched and narrower when squashed (i.e. they have a negative 'Poisson's ratio'). This property results in many beneficial effects in the characteristics of the system that make auxetics superior to conventional systems in many practical and high tech applications, including aeronautics where, for example, auxetics are being proposed as potential components for the production of better quality lifting devices such as helicopter rotor blades or airplane wings. This work reviews and discusses the behaviour of known and novel cellular systems, which exhibit this unusual but highly desirable property.

Keywords: auxetic, negative Poisson's ratios, cellular, honeycombs, mechanical properties.

INTRODUCTION

Auxetic materials exhibit the unexpected property of becoming wider when stretched and narrower when squashed [1], that is they have a negative Poisson's ratio (fig. 1).

This unusual behaviour is not commonly observed in materials that are normally employed in everyday life. In fact, although it is has been known for a long time that that negative Poisson's ratios could theoretically exist (the classical theory of elasticity states that it is possible for isotropic three dimensional materials to exhibit Poisson's ratios in the range $-1 \leq \nu \leq +0.5$) until very recently, the prospect of making use of such materials on a large scale was not researched. In fact, when negative Poisson's ratios were first reported for single crystalline iron pyrites in the first half of the 20th century, it was attributed to twinning defects and regarded as an anomaly [2].

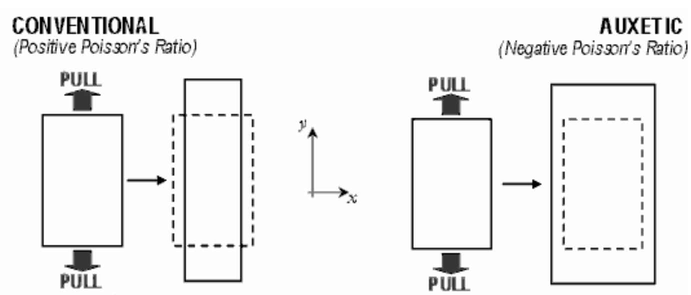

Fig. 1: Auxetic vs. conventional behaviour

However, in the late 1980's, the study of materials exhibiting negative Poisson's ratios became more established and since then, negative Poisson's ratios have been predicted, discovered or deliberately introduced in several classes of naturally occurring and man-made materials including foams [3–9], polymers [1, 10–14], composites [15, 16], gels [17, 18], laminates [19], metals [20], silicates [21 –25] and zeolites [26, 27]. Furthermore, various two- and three-dimensional theoretical models and structures that can lead to negative Poisson's ratio have also been proposed. These include two- and three-dimensional 're-entrant' systems [1, 28–32], models based on rigid 'free' molecules [33–35] and chiral structures [9, 36-38].

CELLULAR AUXETICS

The term 'cellular solids' relates to systems that 'are made up of an interconnected network of solid struts or plates, which form the edges and faces of cells' [39]. Cellular materials find a number of applications in everyday life ranging from construction applications, where they provide structural support, thermal insulation, etc. to specific applications where they are used as filters, to provide buoyancy (due to their low density, their floating potential is pronounced), etc. Their widespread use arises from the fact that cellular solids exhibit a number of physical characteristics that make them superior to other classes of materials. For example, cellular solids benefit from low thermal mass, which increases the insulation efficiency since less heat has to be provided or abstracted from them. They also benefit from being able to sustain large compressive strains, absorb considerable amounts of energy without creating high stress, etc.

In recent years, a considerable amount of work has been done on cellular solids that exhibit auxetic behaviour. Negative Poisson's ratios impart on the cellular systems various additional enhanced characteristics that make them behave even better than conventional cellular solids. For example, when compared to conventional cellular solids, cellular auxetics have been found to be able to decrease more efficiently the propagation of vibrations [40], exhibit enhanced indentation resistance [41] and exhibit an increase in bending stiffness of constructional elements and augmented shear resistance [42].

Such properties of cellular solids, and in particular, those of auxetic cellular solids can be extremely useful in various high-tech applications, in particular in the aeronautic and space industry. For example, in the aeronautic industry, the use of sandwich structures with an auxetic foam core in the construction of the body of the aircraft is likely to result in a significant decrease in the level of noise that reaches the inside of the cabin with the result of making the atmosphere inside the aircraft more pleasant by reducing noise pollution. (The acoustic absorption properties of auxetics have been shown to be superior to those of conventional materials [3].) Also, cellular auxetics can be used to replace specific components so as to produce higher quality and more reliable equipment. For example, the ability of auxetics to act as smart filters, where their pores can be 'opened up' through the simple procedure of mechanically stretching the filter in one direction, could provide filters that can be easily unclogged and hence do not need replacement, a feature that is highly desirable in the space industry where the availability of 'spare parts' is highly limited.

In cellular solids, like other classes of auxetics, the presence of negative Poisson's ratios requires the cells to have a 'suitable geometry', which deforms through some particular deformation mechanism that causes the 'geometry' to open up. For example, a considerable amount of progress has been made in the modelling, manufacturing and testing of auxetics that contain 're-entrant' units. Referring to fig. 2, it has been shown that a two-dimensional re-entrant cellular system deforming by hinging (changes in the angle θ) exhibits negative Poisson's ratios, the magnitudes of which depend on the geometric parameters h/l and θ [30]. (For this cellular structure to exhibit auxetic behaviour through hinging the angle θ must be between 0^c and $\frac{1}{2}\pi^c$, since if the angle is larger than $\frac{1}{2}\pi^c$, the structure will revert back to a conventional honeycomb and have a positive Poisson's ratio.) Furthermore, it has also been shown that this re-entrant honeycomb exhibits auxetic behaviour when deforming through flexure of the ribs [28, 30]. From a structural and commercial point of view, this is very important as flexing structures are easier and less expensive to produce than the hinging structures.

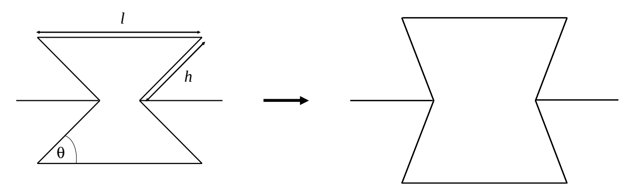

Fig. 2: The 're-entrant hexagonal honeycomb' that exhibits negative Poisson's ratios due to a hinging (i.e. change in θ) mechanism. Note that this structure could also exhibit auxetic behaviour if it deforms through flexure of the ribs

Another class of systems that are known to exhibit negative Poisson's ratios are 'structures' made from rigid units, which 'open up' when loaded in tension through a mechanism involving relative rotation of the rigid units [13,14,26,27]. For example, it is known that systems composed of hinging triangles can exhibit auxetic properties and it has been shown that when equilateral triangles of any size are connected together through hinges at their vertices as illustrated in fig. 3 will exhibit Poisson's ratios of –1 for loading in any direction in the plane of the structure [27].

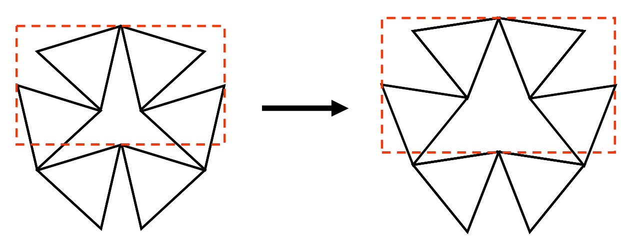

Fig. 3: The 'rotating hinged triangles' structure that exhibits Poisson's ratios of –1 when it deforms through relative rotation of the triangles

Mechanisms involving 'rotating rigid units' are particularly efficient in generating negative Poisson's ratios and it has been shown that they are responsible for negative Poisson's ratios in various classes of materials, and are exhibited even at the molecular level. For example, negative Poisson's ratios in the zeolites THO and NAT can be explained in terms of 'rotating squares', whilst negative Poisson's ratios in the zeolites JBW and ABW can be explained in terms of 'rotating triangles' [26].

In view of all this, this paper examines (through finite elements modelling (FEM) and analytical modelling) the potential of a novel type of cellular systems to exhibit negative Poisson's ratios. This new system is illustrated in fig. 4 and is geometrically similar to a configuration of the 'rotating hinged triangles' where the triangles are at an angle of $\frac{1}{3}\pi^{c}$ to each other. However, unlike the 'rotating hinged triangles' structure, this system does not contain any hinges and instead, it is constructed from beam-like elements of homogeneous material 'welded' together.

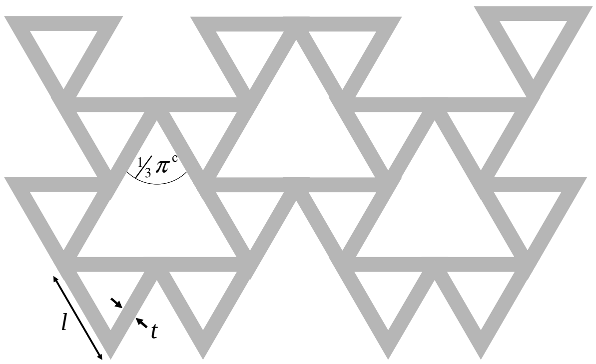

Fig. 4: The geometry of the new 'welded' triangles structure

MODELING THE NEW CELLULAR STRUCTURE

The newly proposed system (see fig. 4) can be considered as a tessellate of two equilateral triangles at an angle of $\frac{1}{3}\pi^{c}$ to each other, which when tessellated in 2D space produce an infinitely large structure. It will be assumed that the triangles are constructed from a homogeneous material with intrinsic Young's modulus E_s and have physical dimensions l (the lengths of the sides of the triangles), t (the thickness in the plane of the structure of beam elements used in the construction) and z (the thickness in the third direction of beam elements).

Using finite-elements modelling (FEM) the effect of uniaxial loading on the 'welded triangles' cellular system illustrated in fig. 5(a) was simulated. This may be considered as a finite but representative section of a larger 2D cellular system. In the simulation, the system was constructed in ANSYS (release 5.7.1) using beam elements, and was solved non-linearly for deformations in the X-direction, which correspond to 10% strain in the X-direction. As illustrated in fig. 5, this simulation suggests that the strain in the X-direction forced the cellular system to become larger in the Y-direction in such a way that the system maintains its aspect ratio (i.e. a strain of 10% in the X-direction generated an approximately equal strain in the Y-direction, see fig. 5(c)). This suggests that this system exhibits negative Poisson's ratio ν_{xy} equal to *c.* –1. Furthermore, this simulation indicates that the negative Poisson's ratio arises from deformations in the geometry of the triangles, which flex in a highly symmetric manner that has the net effect of making the triangles take a new shape that increases the angle between the triangles.

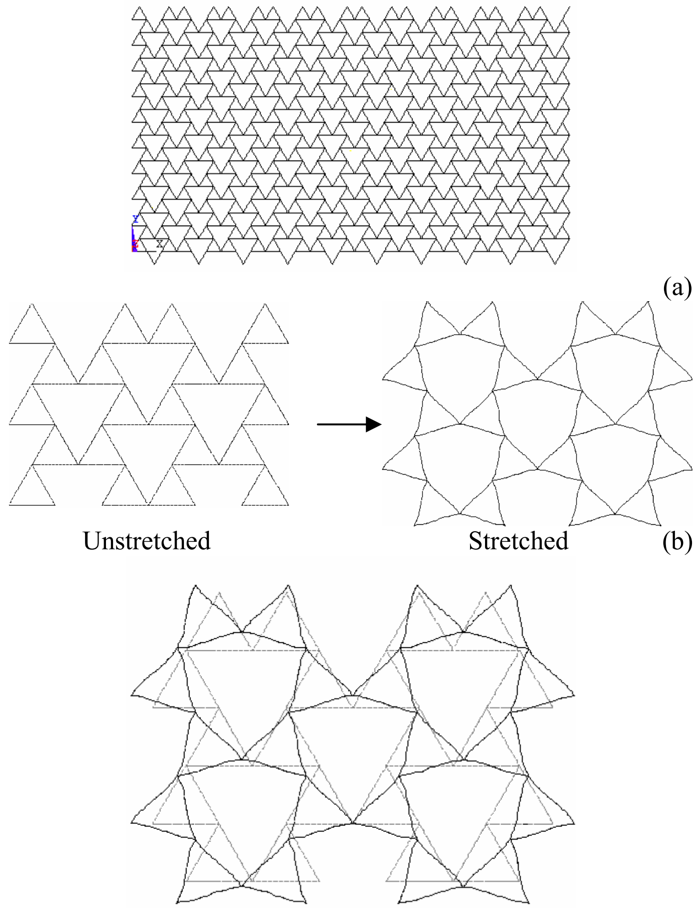

Fig. 5: (a) The 'unstretched' system constructed in ANSYS; (b) part of the system simulated in ANSYS in the 'unstretched' and 'stretched' (10% strain in the X-direction) form; and (c) a 'to-scale' superimposition of (b) to illustrate that the two systems have the same aspect ratios.

To obtain a clearer insight into the behaviour of this system, the analytical expressions for the Poisson's ratios and Young's moduli for such systems made from beams with any dimensions l, t, z) will be derived.

As illustrated in Fig. 6, the minimal unit cell of the structure contains two triangles residing in the rhombus DEFG of side length L_o and unit cell angle of $\frac{1}{3}\pi^{c}$, where L_o is given by:

$$L_o = \sqrt{3}l \,. \qquad (1)$$

However, in this derivation a larger unit cell, GHIJ, will be considered as this unit cell has a rectangular shape and hence the Poisson's ratios may be more easily defined with respect to it. For this 'larger' rectangular unit cell, X_1 and X_2, the dimensions of the unit cell in the Ox_1 and Ox_2 directions respectively, (which correspond to the X- and Y-directions used in FEM), of the undeformed (initial) structure, are given by:

$$X_1^{\text{Init}} = \sqrt{3}L_o \qquad (2)$$

$$\text{and } X_2^{\text{Init}} = L_o \tag{3}$$

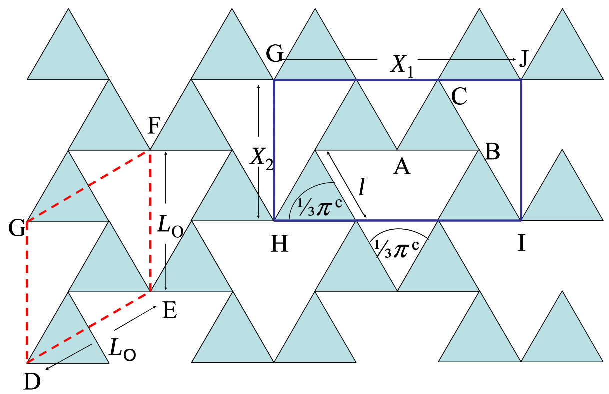

Fig. 6: The unit cell GHIJ used in the analytical model and its relationship with the 'minimal' unit cell DEFG

If the structure is loaded in an Ox_i ($i = 1,2$) direction, moments will be generated, which result in bending (flexing) of the rods as illustrated in fig. 7. Assuming that the deflection is small enough, the deformed structure can be approximated by equilateral triangles, of length l at an angle ω to each other, where ω relates to the angle of deflection ϕ by,

$$\omega = \tfrac{1}{3}\pi + 2\phi \,. \tag{4}$$

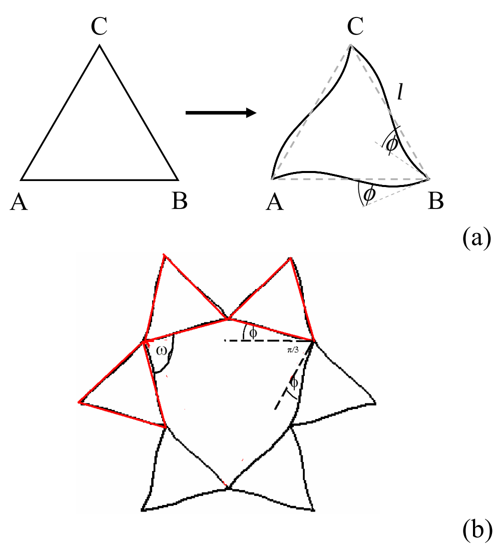

Fig. 7: (a) The changes in shape of the triangles due to loads; (b) the relationship between ω and ϕ.

In such scenario, the lengths X_1 and X_2 of the deformed unit cell are given respectively by,

$$X_1^{\text{Fin}} = \sqrt{3}L(\omega) \tag{5}$$

$$\text{and } X_2^{\text{Fin}} = L(\omega)\,, \tag{6}$$

where,

$$L(\omega) = \sqrt{2}\,l\sqrt{1-\cos\left(\tfrac{1}{3}\pi + \omega\right)}\,. \tag{7}$$

(a) The Poisson's ratios

The engineering Poisson's ratio for loading in the Ox_i direction, is defined by,

$$\nu_{ij} = -\frac{e_j}{e_i}\,. \tag{8}$$

where e_i is the engineering strain in the Ox_i direction, defined by,

$$e_i = \frac{\Delta X_i}{X_i} = \frac{X_i^{\text{Fin}} - X_i^{\text{Init}}}{X_i^{\text{Init}}} = \frac{X_i^{\text{Fin}}}{X_i^{\text{Init}}} - 1\,. \tag{9}$$

In this case the strains are given by:

$$e_1 = \frac{\sqrt{3}L(\omega)}{\sqrt{3}L_o} - 1 = \frac{L(\omega)}{L_o} - 1\,, \tag{10}$$

and

$$e_2 = \frac{L(\omega)}{L_o} - 1\,. \tag{11}$$

Hence the engineering Poisson's ratio for the structure deforming through flexure is given by,

$$\nu_{12} = (\nu_{21})^{-1} = -\frac{e_2}{e_1} = -1\,, \tag{12}$$

(b) The Young's moduli

In analogy with [36], the Young's moduli may be determined through a conservation of energy method. The strain energy per unit volume is given by:

$$U = \frac{1}{2}E_i e_i^2\,. \tag{13}$$

where using Eqs. 1-7, 10 and 11:

$$e_1 = e_2 = \frac{\sqrt{2}\sqrt{1-\cos\left(\tfrac{1}{3}\pi + \omega\right)} - \sqrt{3}}{\sqrt{3}}\,. \tag{14}$$

Taylor expanding $\sqrt{1-\cos\left(\frac{1}{3}\pi+\omega\right)}$ about $\frac{2}{3}\pi^{c}$ gives,

$$\sqrt{1-\cos\left(\tfrac{1}{3}\pi+\omega\right)} = \sqrt{1-\cos\left(\tfrac{2}{3}\pi\right)}+\sqrt{2}\cos\left(\frac{2\pi}{6}\right)\phi. \tag{15}$$

In this way it is possible to approximate Eq. 14 as,

$$e_1 = e_2 = \frac{\phi}{\sqrt{3}}\,, \tag{16}$$

while Eq. 13 becomes,

$$U = \frac{1}{2}E_i\frac{\phi^2}{3} = \frac{1}{6}E_i\phi^2\,. \tag{17}$$

This strain energy is stored as bending energy in the bent ribs of the structure. There are twelve rods per unit cell (four triangles each containing three rods), and hence the bending energy per unit cell is given by:

$$W = 12U_{\text{Rod}}\,, \tag{18}$$

where U_{Rod} is the bending energy of a single rod. From standard beam theory, this bending energy for one rib is given by:

$$U_{\text{Rod}} = 2\int_0^{\phi} M(\phi)\,d\phi\,, \tag{19}$$

where the factor of 2 arises because there is a torque acting at both ends of the rods.

Due to the symmetric nature of the system, the moment applied at each point will be distributed evenly over the four vertices, resulting in a symmetric bending of the rod as can be seen from fig. 5(b) (which shows a representative part of the 'unstretched' and 'stretched' systems). Hence if the applied moment at each junction is $4M$, that on each side of a single beam will be M. This allows the determination of the angular deflection, ϕ, using the standard beam theory [43] and involving the principle of superimposition:

$$\phi = \frac{Ml}{3E_sI} - \frac{Ml}{6E_sI} = \frac{Ml}{6E_sI}\,, \tag{20}$$

where I is the second moment of the area about the neutral axis, which in this case is given by:

$$I = \frac{1}{12}t^3z\,. \tag{21}$$

Substituting Eq. 21 into Eq. 20, yields,

$$\phi = \frac{Ml}{6E_sI} = \frac{2Ml}{E_st^3z}\,, \tag{22}$$

and putting M subject of the formula, an expression for M as a function of ϕ is obtained:

$$M(\phi) = \frac{E_st^3z}{2l}\phi\,, \tag{23}$$

Thus, by substituting Eq. 23 into Eq. 19 and integrating, the bending energy in each rib can be determined to be,

$$U_{\text{Rod}} = \frac{E_st^3z}{2l}\phi^2\,, \tag{24}$$

But from the principle of conservation of energy, the bending energies stored in the rods (Eq. 18) is related to the strain energy per unit volume U through:

$$U = \frac{1}{V}W\,, \tag{25}$$

where V is the volume of the unit cell, which in this case may be approximated by:

$$V = X_1X_2z \approx \left(\sqrt{3}L_o\right)\left(L_o\right)z = 3\sqrt{3}l^2z\,, \tag{26}$$

In this way using Eqs. 17, 18 and 24-26, after rearranging and putting E_i as subject of the formula, the following expression is obtained:

$$E_i = 4\sqrt{3}E_s\left(\frac{t}{l}\right)^3. \tag{27}$$

(c) Summary

To summarise, the Poisson's ratios and Young's moduli for this cellular structure deforming *via* flexure are given by,

$$\nu_{12} = \nu_{21} = -1 \tag{12}$$

$$\text{and } E_i = 4\sqrt{3}E_s\left(\frac{t}{l}\right)^3. \tag{27}$$

Note that the results in Eq. 12 agrees with the results of the finite element modelling.

DISCUSSION AND CONCLUSIONS

Through this work it has been shown that cellular structures made from beams 'welded' together in such a way that the resulting system contains equilateral triangles exhibits negative Poisson's ratios.

In particular, it has been shown through analytical modelling that whilst the Poisson's ratios for such systems are always equal to –1 (provided the triangles are equilateral), the stiffness (Young's moduli) depends on the lengths *l* of sides of the triangles (i.e. the length of the rods), the thickness of the beams and the intrinsic Young's modulus of the material used in manufacturing the cellular structure. In particular the Young's moduli increase as the length *l* decreases and the thickens *t* of the beam increases. This is very significant as it suggests that this is an auxetic structure that can be tailor made to exhibit any magnitudes of stiffness without affecting the Poisson's ratio.

It should be noted that in reality if the values of *t* and *l* are such that the stiffness of the structure becomes too high, other deformation mechanisms could start to compete with the deformation mechanism proposed here (flexure of the sides of the triangles). This would have an effect on the magnitude of the Poison's ratios and probably decrease the auxeticity.

It should also be noted that the cellular structure is particularly interesting as its shape is such that loading in a unaxial direction results in an increase in the empty space between the triangles in a highly symmetric fashion where, to a first approximation, the large equilateral triangles start to become like 'regular hexagons' (fig. 7 (b)). Such a property is highly desirable in filters and hence this newly proposed cellular structure is particular suitable for use as a smart filter where the porosity can be controlled through the application of uniaxial strains.

Given the commercial importance of systems with negative Poisson's ratio, and the simplicity of this structure, we hope that this work will be exploited in the near future so as to commercially produce new auxetic cellular systems that will be able to benefit from so many enhanced characteristics.

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