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On the properties of real finite-sized planar and tubular stent-like auxetic structures

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Auxetics, i.e. systems with a negative Poisson's ratio, exhibit the unexpected property of becoming wider when stretched and narrower when compressed. This property arises from the manner in which the internal geometric units within the system deform when the system is submitted to a stress and may be explained in terms of 'geometry–deformation mechanism' based models. This work considers realistic finite implementations of the well known rotating squares system in the form of (i) a finite planar structure and (ii) a tubular conformation, as one typically finds in stents. It shows that although the existing models of the Poisson's ratios and moduli based on periodic systems may be appropriate to model systems where the geometry/deformation mechanism operate at the micro- or nano- (molecular) level where a system may be considered as a *quasi* infinite system, corrections to the model may need to be made when one considers finite structures with a small number of repeat units and suggests that for finite systems, especially for the 2D systems, the moduli as predicted by the periodic model may be significantly overestimating the moduli of the real system, even sometimes by as much as 200%.

1 Introduction The past three decades have been characterised by some significant important developments in the field of auxetics [1], i.e. materials and structures having a negative Poisson's ratio which get wider rather than thinner when uniaxially stretched [1–24]. Through the work of several scientists and engineers, who performed much of the important early modelling and experimental work on auxetics, we now have a much better understanding of what gives rise to this anomalous behaviour, and how this property may be exploited in practical applications to give rise to superior products [22–37]. It is beyond the scope of this work to review all the recent discoveries made on auxetics, but it seems that it is now widely accepted that:

(i) auxetic behaviour may normally be explained in terms of particular geometric features in the internal structure of the system which deform when the system is subjected to a uniaxial stress (the intricate interplay between the 'geometry and the deformation mechanism');

(ii) auxetic behaviour is a scale-independent property, and thus, the same geometry/deformation mechanism may operate at the macro-, micro- and nano- (molecular) level.

As a result of this, it is becoming common practice to study auxetic materials and structures in terms of two and three-dimensional geometry–deformation mechanism model systems which can exhibit negative Poisson's ratio. Such models range from the well-studied 're-entrant' systems [1, 9–11, 38] to models based on rigid 'free' molecules [3–8], chiral structures [12–14] and models based on rotating rigid units [21–24]. These models have been used to explain the negative Poisson's ratio behaviour present in various classes of materials such as auxetic foams [2], silicates or zeolites [15–16].

Whilst it is fairly obvious that auxeticity at the nano- or micro-level will result in the production of new materials having significantly improved properties over their conventional counterparts, it is becoming more evident that auxetic macrostructures have their own niche range of applications. Such systems are likely to be much easier to manufacture from readily available conventional materials and still benefit from the enhanced properties that result from having a negative Poisson's ratio. For example, recent work has shown that the rotating squares model originally proposed by Grima and others [39, 40] lends itself well to use at the macro-scale and it has been demonstrated that macro-systems made from the 'rotating squares' configuration may find applications in the construction of sophisticated products such as oesophageal stents [41–43] to more mundane ones used in the construction of deployable furniture and other household items [44]. Stents, which may be described as scaffolds implanted in constricted sections of body vessels in order to keep them open, have been widely studied in terms of structure, fluid interaction [45, 46] and coatings for drug release [47].

An important difference between auxetic materials and macro-scale auxetics is that whilst at the nano- or micro-scale, the systems may be considered as *quasi* infinite systems, auxetic macrostructures are normally finite structures having a small number of repeat units. Such rigid unit systems may not be adequately modelled by existing theoretical models such as those derived by Grima et al. [21–24] which are based on the use of periodic boundary conditions so as to simulate an infinitely large system.

In view of all this, here we re-examine the properties of a typical planar two-dimensional system constructed from rotating rigid units (rigid squares connected through hinges) where we consider the properties of systems which are made from a finite limited number of sub-units rather than a periodic model. We also examine the properties of a three-dimensional tubular systems, such as the ones which can be employed for stent design and manufacture.

2 Analytical modelling

2.1 Two-dimensional finite rotating squares model

In this paper we shall be first deriving analytical expressions for the 3 × 3 compliance matrix **S** of finite systems made from connected squares conformations having N_1 squares in the Ox_1 direction and N_2 squares in the Ox_2 direction.

As in the case of the periodic model [21], we shall consider a two-dimensional structure built with perfectly rigid squares of side length a, hinged at their corners and aligned in the Ox_1–Ox_2 plane as shown in Fig. 1a. We shall assume that the angle between two such squares is θ and that the stiffness in the structure is solely imparted by the stiffness of the hinges which connect the adjacent squares, in particular, a stiffness which opposes changes in the angles θ. It shall be assumed that the hinges satisfy the equation:

$$M = K_\mathrm{h}(\delta\theta), \tag{1}$$

where M is the moment applied to the squares, $\delta\theta$ is the angular displacement due to M, and K_h is the spring constant for the hinge.

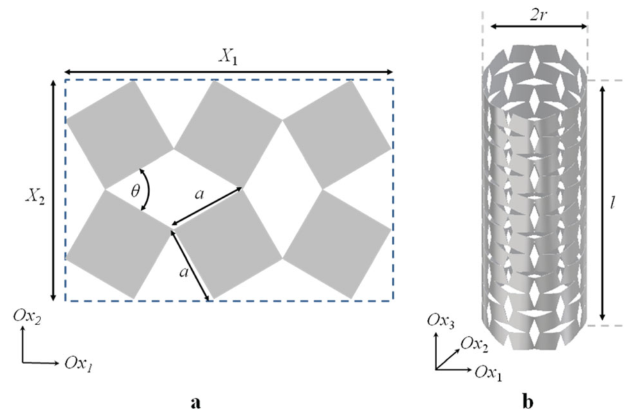

Figure 1 (a) 2D planar finite $N_1 \times N_2 = 3 \times 2$ system composed of rotating squares of size a. (b) 3D tubular system composed of $N_1 \times N_2 = 12 \times 12$ squares.

a. The on-axis Poisson's ratio

Referring to Fig. 1a, it may be shown that for a system having $N_1 \times N_2$ squares, the dimensions of the structure in the Ox_i directions are given by:

$$X_i = N_i a\left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right], \quad i = 1, 2, \tag{2}$$

where since we are assuming that the structure deforms solely by relative rotation of the squares, then the geometric parameter a may be considered as a constant and hence X_i are functions of the single variable θ, i.e. X_i=$X_i(\theta)$. As a result of this, Poisson's ratios ν_{ij} in the Ox_i–Ox_j plane for loading in the Ox_i direction may be computed by:

$$\nu_{21} = (\nu_{12})^{-1} = -\frac{\mathrm{d}\varepsilon_1}{\mathrm{d}\varepsilon_2} = -\frac{\mathrm{d}X_1/X_1}{\mathrm{d}X_2/X_2} = -\frac{\mathrm{d}X_1/\mathrm{d}\theta}{\mathrm{d}X_2/\mathrm{d}\theta}\frac{X_2}{X_1}, \tag{3}$$

i.e. from Eqs. (2) to (3), the on-axis Poisson's ratios simplify to:

$$\nu_{21} = \nu_{12} = -1, \tag{4}$$

as was the case for the periodic model which is meant to represent an infinite system.

b. The on-axis Young's moduli

As in the case of the model of the infinite periodic system, the Young's moduli of the finite systems may be derived using the conservation of energy approach.

Recognising that for a system having N_1 by N_2 squares, there will be $2N_1N_2$–N_1–N_2 hinges, then the work done by the system due to the changes in the inter-square angles from θ to $\theta + \mathrm{d}\theta$ that accompany a small strain is given by:

$$W = (2N_1N_2 - N_1 - N_2)\left[\frac{1}{2}K_\mathrm{h}(\mathrm{d}\theta)^2\right], \tag{5}$$

where K_h is the stiffness constant of the hinges as defined through Eq. (1). Thus, the strain energy per unit volume of the structure is given by:

$$U = \frac{1}{V} W, \tag{6}$$

where V is the volume of the structure including the void space between the squares given by (assuming a small thickness t in the third dimension):

$$V = X_1 X_2 t. \tag{7}$$

Also, from the principle of conservation of energy and since $X_i = X_i(\theta)$, the strain energy per unit volume of the structure due to an infinitesimally small strain $d\varepsilon_i$ for loading in the Ox_i direction ($i = 1,2$) is given by:

$$U = \frac{1}{2} E_i (d\varepsilon_i)^2 = \frac{1}{2} E_i \left(\frac{dX_i}{X_i} \right)^2 = \frac{1}{2} E_i \left(\frac{1}{X_i} \frac{dX_i}{d\theta} \right)^2 (d\theta)^2. \tag{8}$$

Thus from Eqs. (5) to (8) we have:

$$\frac{1}{2} E_i \left(\frac{1}{X_i} \frac{dX_i}{d\theta} \right)^2 (d\theta)^2 = \frac{1}{X_1 X_2 t} (2N_1 N_2 - N_1 - N_2) \times \left[\frac{1}{2} K_h (d\theta)^2 \right] \tag{9}$$

and hence the Young's moduli E_i ($i = 1, 2$) are given by:

$$E_i = \frac{X_i^2}{X_1 X_2 t} (2N_1 N_2 - N_1 - N_2) K_h \left(\frac{dX_i}{d\theta} \right)^{-2}. \tag{10}$$

For loading in the Ox_1 direction:

$$E_1 = \frac{X_1}{X_2 t} (2N_1 N_2 - N_1 - N_2) K_h \left(\frac{dX_1}{d\theta} \right)^{-2}, \tag{11}$$

i.e.:

$$E_1 = \frac{K_h}{N_1 N_2 t} (2N_1 N_2 - N_1 - N_2) \left(\frac{a}{2} \left[\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right) \right] \right)^{-2} = \frac{1}{2N_1 N_2 t} (2N_1 N_2 - N_1 - N_2) \left[\frac{8K_h}{a^2} \frac{1}{1 - \sin(\theta)} \right], \tag{12}$$

i.e.:

$$E_1 = \frac{1}{t} \left[1 - \frac{1}{2} \left(\frac{1}{N_1} + \frac{1}{N_2} \right) \right] E^{inf}, \tag{13}$$

where E^{inf} is the Young's modulus for the equivalent infinite periodic model having a unit thickness in the third dimension as derived by Grima et al. [39] given by:

$$E^{inf} = \frac{8K_h}{a^2} \frac{1}{1 - \sin\theta}. \tag{14}$$

Similarly, for loading in the Ox_2 direction,

$$E_2 = \frac{1}{t} \left[1 - \frac{1}{2} \left(\frac{1}{N_1} + \frac{1}{N_2} \right) \right] E^{inf}. \tag{15}$$

In the particular case when $N_1 = N_2 = N$, the Young's moduli simplify to:

$$E_i = \frac{1}{t} \left(1 - \frac{1}{N} \right) E^{inf}. \tag{16}$$

These equations for the Poisson's ratios and Young's moduli for both the general and particular case of $N_1 = N_2$ satisfy the thermodynamic requirements given by:

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}, \quad |\nu_{ij}| \leq \sqrt{\frac{E_i}{E_j}}. \tag{17}$$

Furthermore, it should be noted that as $N \to \infty$, $(1 - (1/N)) \to 1$ and for a system with unit thickness, $E_i \to E_i^{inf}$ as expected.

c. The full compliance matrix

For N_1, $N_2 > 1$, the rigidity of the squares results in a structure which is geometrically constrained not to shear. As a result, the system will have values of zero for the five elements of the compliance matrix which are associated with shearing and hence the compliance matrix for this system is of the form:

$$\boldsymbol{S} = [s_{ij}] = \frac{1}{E} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{18}$$

where

$$E = E_1 = E_2. \tag{19}$$

2.2 Three-dimensional tubular structure constructed from rotating squares If a tube had to be constructed from $N_1 \times N_2$ squares such that the N_1 squares are aligned along the circumference of the tube and N_2 squares along the length (Fig. 1b), it should be emphasised that the aspect ratio and dimensions of the tube would be dependent on the size of the squares, the angles between them and the number of squares used. In particular, to ensure proper connectivity, the radius r of the tube must be related to a, θ and N_1 through:

$$2\pi r = N_1 a \left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right) \right], \quad \frac{N_1}{2} \in \mathbb{Z}^+, \tag{20}$$

whilst the length of the tube l must be equal to:

$$l = N_2 a\left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right], \quad N_2 \in \mathbb{Z}^+. \tag{21}$$

Furthermore, in such cases one must also define the alignment of the tube in 3D space as well as the manner how the Poisson's ratio is defined. In this case, it shall be assumed that the length of the tube is aligned parallel to the Ox_3-direction whilst the circular cross-section is parallel to the Ox_1–Ox_2 plane.

a. Properties for loading along the length of the tube
For such systems, one may define the Poisson's ratio for stretching along the length through:

$$\nu_{31} = \nu_{32} = \nu_{lr} = -\frac{d\varepsilon_r}{d\varepsilon_l} = -\frac{l}{r}\left(\frac{dr}{d\theta}\right)\left(\frac{dl}{d\theta}\right)^{-1} = -1, \tag{22}$$

whilst the stiffness for pulling along the length may be estimated through a Young's modulus equivalent to E_3 which considers the structure as a hollow tube, in analogy to what one normally assumes when studying carbon nanotubes [48, 49], and may be derived through a conservation of energy approach as above.

Recognising that for a tube having $N_1 \times N_2$ squares such that the N_1 squares are aligned along the circumference of the tube and N_2 squares along the length, there will be $2N_1N_2$–N_1 hinges, then the work done by the system due to the changes in the inter-square angles from θ to $\theta + d\theta$ that accompany a small strain is given by a modified form of Eq. (5) above:

$$W = (2N_1N_2 - N_1)\left[\frac{1}{2}K_h(d\theta)^2\right]. \tag{23}$$

Thus, the strain energy per unit volume of the structure is given by Eq. (6) where in this case, it would be appropriate to consider V as the volume of the hollow tube, i.e. assuming that the tube has a small thickness t of the solid portion, then the volume of tube (i.e. the squares and the empty spaces between them along the surface) may be approximated by:

$$V = 2\pi r l t = N_1 N_2 a^2 t\left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right]^2. \tag{24}$$

Also, from the principle of conservation of energy and since, $l = l(\theta)$ the strain energy per unit volume of the structure due to an infinitesimally small strain $d\varepsilon_l$ for loading in the Ox_3-direction is given by:

$$U = \frac{1}{2}E_3(d\varepsilon_l)^2 = \frac{1}{2}E_3\left(\frac{dl}{l}\right)^2 = \frac{1}{2}E_3\left(\frac{1}{l}\frac{dl}{d\theta}\right)^2(d\theta)^2. \tag{25}$$

Thus from Eq. (6) and Eqs. (23) to (25) we have:

$$E_3 = K_h \frac{(2N_1N_2 - N_1)}{N_1N_2a^2t\left[\cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\right]^2}\left(\frac{1}{l}\frac{dl}{d\theta}\right)^{-2}, \tag{26}$$

i.e.:

$$\begin{aligned} E_3 &= K_h\frac{(2N_1N_2 - N_1)}{N_1N_2a^2t}\left[\cos\left(\frac{\theta}{2}\right) - \sin\left(\frac{\theta}{2}\right)\right]^{-2} \\ &= K_h\frac{4(2N_1N_2 - N_1)}{N_1N_2a^2t[1 - \sin(\theta)]} \\ &= \frac{1}{t}\left(1 - \frac{1}{2N_2}\right)E^{\text{inf}}, \end{aligned} \tag{27}$$

where E^{inf} is the Young's modulus for the equivalent infinite periodic model having a unit thickness in the third dimension as derived by Grima et al. [39] given by Eq. (14).

b. Properties for loading in a radial direction
If one had to consider the properties for loading in a radial direction, one would need to recognise that the properties would depend on the manner how the tube flattens upon compression. This property would depend on the out-of-plane properties of the system, including the thickness of material. However, it should be noted that one could also consider a scenario where the cross-section is constrained to remain circular. This scenario is not dissimilar to what one observes when tubular stent devices are opened up after insertion, since these are inflated from the inside to assume the more open conformation. Thus, given that the system being studied here is being considered for use in the medical field for the manufacture of oesophageal stents, it would be useful to derive the properties of a system which is loaded in the radial direction whilst retaining a circular cross-section.

In such systems constrained to retain a circular cross-section, by definition, the Poisson's ratios in planes perpendicular to the length of the tube would be -1, i.e.:

$$\nu_{21} = \nu_{12} = -1, \tag{28}$$

as is the Poisson's ratio for loading in a radial direction and measuring the Poisson's ratio in a plane which is perpendicular to the cross-section since:

$$\begin{aligned} \nu_{13} = \nu_{23} = \nu_{rl} = -\frac{\varepsilon_l}{\varepsilon_r} = -\frac{r}{l}\left(\frac{dl}{d\theta}\right)\left(\frac{dr}{d\theta}\right)^{-1} \\ = -1. \end{aligned} \tag{29}$$

For such systems retaining a circular cross-section, the stiffness for pulling along a radial direction may also be estimated through a Young's modulus equivalent to E_1 and E_2 which considers the structure as a hollow tube through a conservation of energy approach as above where in this case,

$$E_1 = E_2 = E_r = K_h \frac{(2N_1N_2 - N_1)}{N_1N_2a^2t[\cos(\theta/2) + \sin(\theta/2)]^2}\left(\frac{1}{r}\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^{-2}, \quad (30)$$

i.e.:

$$E_1 = E_2 = E_3 = E_r = E_l = \frac{1}{t}\left(1 - \frac{1}{2N_2}\right)E^{\inf}. \quad (31)$$

3 Discussion The expressions derived above clearly suggest that the mechanical properties of these systems are in some ways similar to those afforded by a system presenting an infinite tessellation, but differ in some respects.

In particular, the derivations clearly show that whilst the Poisson's ratio of the finite systems are exactly identical to that of the infinite systems, this is not the case with the Young's moduli where the exact value of the modulus is dependent on the size, the number and configuration of the units as well as whether the system is planar or tubular.

In fact, as illustrated in Table 1, which shows how the values of the moduli for a finite two-dimensional system considered here differs from the published model for the infinite system, one may note that, although as N_1 and N_2 increase the two models become almost equivalent, in the case of smaller systems, this difference is very significant. For example, for a system having just 2×2 squares, this difference is such that the modulus of the finite system is only half (0.5) that predicted by the model of the infinite system with the difference becoming one in ten (0.1) for a 10×10 system and one in hundred (0.01) for a 100×100 system. Note that in all cases, the finite system is less stiff than the infinite system. This reduction in stiffness is to be expected since the finite systems contain a lower number of effective hinges when compared to their equivalent fully periodic system and may be considered as an edge effect since it is obvious that the deviations would be much more pronounced for systems which have high 'edge' to 'overall size' ratio.

Likewise, the tubular geometry exhibits similar properties, but in this case, as illustrated in Table 2, which shows how the values of the moduli for a tubular system differs from the published model for the infinite system, we note that the modulus is only dependent on N_2, the number of squares along the length. The two models, i.e. the tubular model and the published one for the infinite system become almost equivalent for long tubes having large values of N_2. Once again, in the case of smaller systems, this difference is very significant and for the shortest possible systems with just one layer of squares (crown) along the length, the modulus would be just half of that predicted by the model of the infinite system.

It should also be emphasised that if one had to compare a tubular system with $N_1 \times N_2$ squares with its equivalent planar system with the same number of squares $N_1 \times N_2$, the tubular system would be stiffer, as expected, since in this case, there would be the additional hinges which join the opposite edges to form the tube. In this direction (along the circumference), the system can be considered as being without ends and thus, the only contributing edge effects to the difference in the Young's modulus would be those arising from the squares at the edges of the tube. Here, it should be noted that the size of the squares along the circumference also has an effect on the Young's modulus of the tube. In particular, if one had to compare two tubes with

Table 1 This table indicates the agreement of the Young's moduli between finite-sized 2D planar systems of differing size and infinitely sized systems. Note that the difference between the finite 2×2 system and the infinitely sized system is of 200% since the Young's modulus of the finite 2×2 system is only half of the infinitely sized system.

N_1	N_2										
	2	3	4	5	6	7	8	9	10	100	1000
2	0.500	0.583	0.625	0.650	0.667	0.679	0.688	0.694	0.700	0.745	0.750
3	0.583	0.667	0.708	0.733	0.750	0.762	0.771	0.778	0.783	0.828	0.833
4	0.625	0.708	0.750	0.775	0.792	0.804	0.813	0.819	0.825	0.870	0.875
5	0.650	0.733	0.775	0.800	0.817	0.829	0.838	0.844	0.850	0.895	0.900
6	0.667	0.750	0.792	0.817	0.833	0.845	0.854	0.861	0.867	0.912	0.916
7	0.679	0.762	0.804	0.829	0.845	0.857	0.866	0.873	0.879	0.924	0.928
8	0.688	0.771	0.813	0.838	0.854	0.866	0.875	0.882	0.888	0.933	0.937
9	0.694	0.778	0.819	0.844	0.861	0.873	0.882	0.889	0.894	0.939	0.944
10	0.700	0.783	0.825	0.850	0.867	0.879	0.888	0.894	0.900	0.945	0.950
100	0.745	0.828	0.870	0.895	0.912	0.924	0.933	0.939	0.945	0.990	0.995
1000	0.750	0.833	0.875	0.900	0.916	0.928	0.937	0.944	0.950	0.995	0.999

Table 2 This table indicates the agreement of the Young's moduli between finite-sized 3D tubular systems of differing size and infinitely sized systems where $(N_1/2) \in \mathbb{Z}^+$.

N_1	N_2										
	2	3	4	5	6	7	8	9	10	100	1000
2, 4, 6, …	0.750	0.833	0.875	0.900	0.917	0.929	0.938	0.944	0.950	0.995	0.9995

the same number of squares along their length and the same radius, such that the number of squares in the circumferential direction is different, the one with the higher number of squares is expected to be stiffer. This is evident from Equation (31), which shows that the Young's modulus for such structures is independent of N_1 and dependent on E^{inf} which according to the analytical model increases as a, the size of the squares, decreases.

All this has some general implications in the construction and modelling of macro-scale auxetic systems which have a finite number of repeat units. Such macro auxetic systems of finite-size may be more commercially viable to manufacture on a large scale when compared to nano-level auxetics and in such systems it is important to understand how the mechanical properties are changing on converting the auxetic system from an infinitely sized one to a finite-sized one, in either a planar or tubular form. The formulas derived above will serve as a useful conversion tool to obtain the properties of such finite-sized auxetic systems.

It should also be noted that the approach used in this paper also sheds light on the work that needs to be done if one had to estimate the properties of finite-sized auxetic systems on changing sample size. Thus, for example, if one had to carry out an experimental investigation on a 3×3 sample system then one would need to make appropriate corrections to the analytical model for estimating the properties of different sized systems (for example a 9×7 system) since the edge effect discussed here would be much more pronounced in smaller systems.

Before we conclude, it should be mentioned that although the studied systems are clearly considering a highly realistic and tangible scenario they may still be considered as idealised in some aspects. For example, in real systems it would be highly improbable that the square units would behave as perfectly rigid units and instead it would have been more realistic to assume some other modes of deformation. In particular, it should be noted that real systems where the squares motif is generated through the use of perforations as discussed by Grima and Gatt [50] and Bertoldi et al. [51], or in stents as discussed by Ali and Rehman [41], Ali et al. [42] and Bhullar et al. [43], then one would expect that there would also be some out-of-plane deformations [50].

These out-of-plane deformations are of particular importance in the case of stents. In fact an analysis of such tubular structures suggests that due to differences in the connectivity of the peripheral and inner crowns, the inner and outer (edge) parts of the stent have a correspondingly different Young's modulus in the radial direction. More specifically, referring to Fig. 2, the inner crowns are connected to two adjacent crowns, while those at the periphery are only connected to one crown. Thus, since the peripheral units have fewer connections, they would be less rigid than the inner units. This means that when the stent is inflated, the peripheral units tend to open up to a higher extent when compared to inner units. This causes the undesirable effect of flaring out of the edges known as dog-boning. Thus, it is important to take such effects into consideration when designing stents, and other finite-sized tubular structures. Here it is important to note that as a continuation of this work, simulations of tubular systems under the influence of a radial pressure can be conducted in order to assess the extent of dog-boning and/or foreshortening. In addition one should also study the effect that the blood flow has on such auxetic stents through dynamic analysis. Such further studies could, for example, follow the methodology employed in published studies relating to the impact assessments of blood flow on stents [46] and fluid auxetic structure interaction [33–35, 37] which have already been studied elsewhere.

Finally, it should be emphasised that what is being discussed here is also likely to apply to a number of other similar model structures although in such cases, the correction factors might be different.

4 Conclusions In this paper we have shown that for finite-sized 2D planar auxetic structures and 3D tubular systems based on the rotating squares system, some

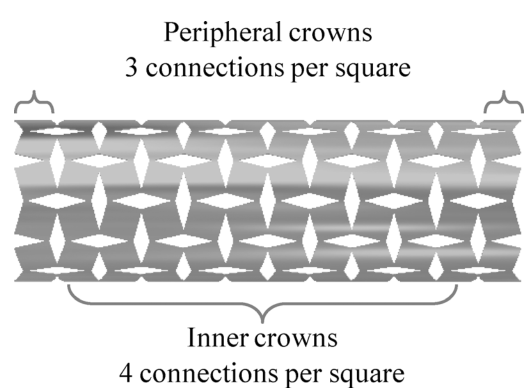

Figure 2 A tubular structure constructed from rotating rigid squares showing the different types of connections for the inner and outer crowns.

mechanical properties will change when compared to the infinitely sized systems. In particular, the Young's moduli for infinitely sized systems may overestimate the Young's moduli for finite-sized systems by up to 200%, something which may be considered as an edge effect. These edge effects are particularly pronounced in the case of 2D systems. In view of the many practical applications that auxetics are likely to find in everyday objects as well as highly sophisticated ones such as oesophageal stents, we are confident that this work will be of use to scientists and manufacturers alike who may wish to produce devices and auxetic components of finite size.

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