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On the behaviour of bi-material strips when subjected to changes in external hydrostatic pressure

Ruben Gatt, Daphne Attard and Joseph N. Grima*

Faculty of Science, University of Malta, Msida MSD 2080, Malta

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Bi-material strips constructed from components having different mechanical properties, in particular different Young's moduli and/or Poisson's ratio, are shown to curve when subjected to changes in the external hydrostatic pressure in a similar manner that such strips curve when subjected to a change in temperature when the component materials have different coefficients of thermal expansion. This behaviour will be particularly pronounced if the material having the lower Young's modulus also exhibits a negative Poisson's ratio (auxetic).

Keywords: Mechanical properties; Thermal expansion; Auxetic; Negative Poisson's ratio; Compressibility

The way that materials change their size in response to changes in temperature and pressure has been the subject of considerable research. In this context, it is important to mention the pioneering work by Bridgman [1,2] on the effect of hydrostatic pressure on properties such as ductility (flow and fracture behaviour) of a variety of monolithic metals. Moreover, Bridgman [1,2] and other workers [2–6] have further investigated the effect of hydrostatic pressure on non-metallic materials including intermetallics, composites, ceramics as well as polymer-based systems. Lewandowski and Lowhaphandu [2] provide an excellent review which discusses the mechanical behaviour of commercially important materials and how they are influenced by the imposed stress state. They outline the fact that cubic metals deform elastically under pressure, while non-cubic metals and heterogeneous materials (such as composites and materials with inclusions) can exhibit plastic deformation if the pressure is high enough.

The extent of which materials change their volume may be described by the volumetric thermal expansion coefficient (α) and the compressibility (β), respectively defined by [7]:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \quad \text{and} \quad \beta = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T. \tag{1}$$

In general, the values of α and β are characteristic of the material and thus different materials will behave differently when subjected to identical environmental conditions. This mismatch of the materials' properties can lead to some very interesting effects. For example, it is well known that a bi-material strip, i.e. a beam constructed from two distinct beams glued together along their lengths in order to form a single strip, adopts a curved configuration when subjected to a change in temperature as illustrated in Figure 1. The radius of curvature κ, for such a beam made from two different materials, "1" and "2", under thermal stress was derived by Timoshenko [8] and is given by:

$$\kappa = \frac{6\left(1+\frac{t_1}{t_2}\right)^2 \mathrm{d}\varepsilon}{3(t_1+t_2)\left[\left(1+\frac{t_1}{t_2}\right)^2 + \left(1+\frac{t_1}{t_2}\frac{E_1}{E_2}\right)\left(\left(\frac{t_1}{t_2}\right)^2 + \frac{t_2}{t_1}\frac{E_2}{E_1}\right)\right]}, \tag{2}$$

where t_i are the thicknesses of the constituent components of the strip, E_i are the Young's moduli of materials "1" and "22", and dε is the difference in the thermal strains experienced by the different components which is given by the term $(\alpha_2 - \alpha_1)\Delta T$. This equation is known to be valid for bi-material strips made from thin ligaments where end effects are neglected [9,10].

Here we highlight the fact that changes in the curvature of bi-material strips do not result solely when such strips are subjected to a change in temperature but may

* Corresponding author. E-mail: joseph.grima@um.edu.mt

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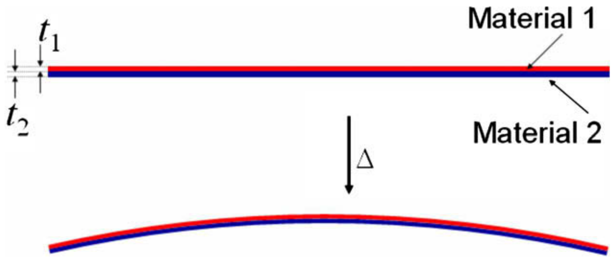

Figure 1. The effect of heat on a bi-material strip made from materials "1" and "2" where the materials have different thermal expansion coefficients α_1 and α_2 where $\alpha_1 > \alpha_2$. Here we are proposing that the same effect may be achieved if the strip is subjected to a change in the external pressure provided that the Young's moduli and/or Poisson's ratio of material "1" are different from those of material "2".

also result from a change in the external pressure provided that the constituent materials have significantly different mechanical properties, in particular different compressibility. This effect arises due to the fact that the constituent parts of a bi-material strip with different mechanical properties will expand at different rates when subjected to a change in external pressure.

To illustrate this, let us consider a cuboidal strip of dimensions xyz made from an isotropic material[1] having compressibility β, Young's modulus E and Poisson's ratio ν. When this strip is subjected to a change $\mathrm{d}P$ in the external pressure, the strip will experience strains along each of the three dimensions of magnitude:

$$\varepsilon = -\frac{1}{3}\beta \mathrm{d}P = \frac{1-2\nu}{E}\mathrm{d}P. \tag{3}$$

Thus, when two isotropic materials "1" and "2" that have different Young's moduli and/or Poisson's ratios are subjected to the same change in external pressure, they will experience different strains, where the difference in strains, $\mathrm{d}\varepsilon$, is given by:

$$\mathrm{d}\varepsilon = \varepsilon_2 - \varepsilon_1 = -\frac{1}{3}(\beta_2 - \beta_1)\mathrm{d}P = \left(\frac{1-2\nu_2}{E_2} - \frac{1-2\nu_1}{E_1}\right)\mathrm{d}P. \tag{4}$$

This difference in strain will result in the curvature of a bi-material strip made from components "1" and "2" when it is subjected to a change in the externally applied pressure. In analogy to the thermal system, and making the same assumptions, the curvature may be obtained by substituting Eq. (4) in Eq. (2), thus suggesting that the extent of curvature will be affected by the difference in the Young's moduli and Poisson's ratio of the two components of the strip and their thickness. In this respect it is interesting to note that if we assume that material "1" has a lower Young's modulus than material "2", then the difference in the strains (which effects the magnitude of the curvature) will be enhanced if the material having the lower modulus also has a negative Poisson's ratio (auxetic [11–18]), whilst the other has a positive Poisson's ratio. For example, if we let $E_1 = \frac{1}{2}E_2$, then the difference in strain (and hence the curvature) will be 14 times more pronounced when $\nu_1 = -1$ and $\nu_2 = +0.3$ when compared to a system with $\nu_1 = \nu_2 = +0.3$. This highlights yet another beneficial property that auxetic materials have when compared to conventional ones [19–22]. Also of interest in this respect are porous materials with closed pores which are known to densify significantly when subjected to high pressures [2].

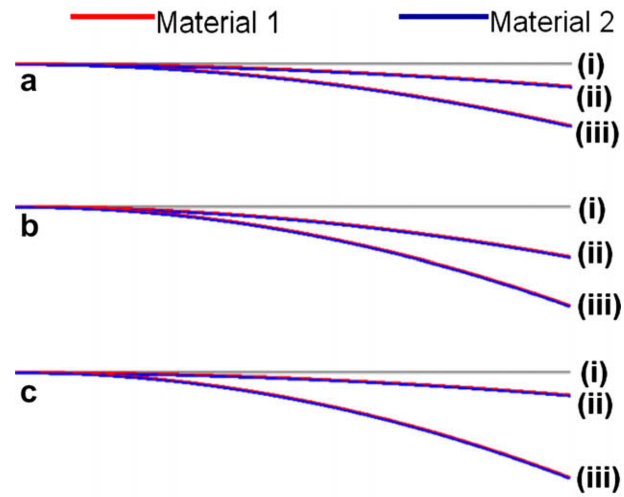

Figure 2. A graphical representation of some of the FEM simulations results obtained for systems subjected to 8000 Pa where (a-i), (b-i) and (c-i) represent the initial systems whilst:
(a-ii) $E_1 = 1\times10^7\,\mathrm{Pa}, \nu_1 = 0.3, E_2 = 2\times10^7\,\mathrm{Pa}, \nu_2 = 0.3$
(a-iii) $E_1 = 0.5\times10^7\,\mathrm{Pa}, \nu_1 = 0.3, E_2 = 2.5\times10^7\,\mathrm{Pa}, \nu_2 = 0.3$
(b-ii) $E_1 = 1\times10^7\,\mathrm{Pa}, \nu_1 = -0.5, E_2 = 1\times10^7\,\mathrm{Pa}, \nu_2 = 0.25$
(b-iii) $E_1 = 1\times10^7\,\mathrm{Pa}, \nu_1 = -0.99, E_2 = 1\times10^7\,\mathrm{Pa}, \nu_2 = 0.49$
(c-ii) $E_1 = 1\times10^7\,\mathrm{Pa}, \nu_1 = 0.3, E_2 = 2\times10^7\,\mathrm{Pa}, \nu_2 = 0.3$
(c-iii) $E_1 = 1\times10^7\,\mathrm{Pa}, \nu_1 = -0.99, E_2 = 2\times10^7\,\mathrm{Pa}, \nu_2 = 0.3$ In all cases the length of the ligaments was set as 20 units whilst the thicknesses were set at $t_1 = t_2 = 0.05$ units. (Note that material "1" is the top part of the bi-material strip whilst materials "2" constitutes the bottom part.)

An analogous model can be derived for bi-material strips made from anisotropic materials. In this case the effect may be more evident due to the wider range of values that the mechanical properties may assume.

The hypotheses presented here are supported through finite element modelling (FEM) simulations which we performed using ANSYS (see Fig. 2) on the two-dimensional equivalents of the bi-material strips.[2] These simulations confirm that bi-material strips with different Young's moduli and/or different Poisson's ratio bend when they are subjected to a hydrostatic pressure, and that the extent of curvature is dependent on the Young's moduli, Poisson's ratio and thickness. They also confirm that auxeticity of the "softer" component may enhance this effect.

It is important to highlight that as a result of the fact that the theory of elasticity is scale independent, the effect described here is also scale independent, i.e. it may be

[1] In this discussion we are assuming that the materials involved behave in an elastic manner, i.e. materials such as cubic metals [2] are used, or the pressure changes are not sufficient to induce plastic deformation. In this respect, it is also important to note that if other types of materials are used (e.g. heterogeneous materials), the materials may deform plastically [2], thus deviating from the model presented here.

[2] Note that for two-dimensional systems, the difference in strains defined by Eq. (4) will be given by $\mathrm{d}\varepsilon = \varepsilon_2 - \varepsilon_1 = \left(\frac{1-\nu_2}{E_2} - \frac{1-\nu_1}{E_1}\right)\mathrm{d}P$.

manifested at any scale of structure. This suggests that it is theoretically possible to make use of the effect described here to construct microstructured materials having components made from bi-material elements which bend when subjected to a change in pressure. Such materials could have very interesting properties, including negative compressibility (negative bulk modulus), i.e. the ability to contract in size when subjected to a change in pressure. In particular, here we note that the foam-type structure constructed from bi-material elements recently proposed by Lakes [23] may in fact also exhibit negative compressibility if the mechanical properties of the two constituent materials are sufficiently different form each other, in particular if the one with the lower Young's modulus is auxetic and the other is conventional, and their relative position is amenable. In this respect we note that the work reported here suggests that the extent of the curvature (which will have an important bearing on the sign of magnitude of the compressibility) can be fine tuned to particular values through careful choice of the materials used and the relative thicknesses of the two strips, thus enabling the construction of systems that are tailor made for specific practical applications.

To conclude, we have shown through simple equations and FEM simulations that bi-material strips with components having different properties will not only curve when subjected to a change in temperature but also when subjected to changes in the external pressure, a property that may have a number of potential applications, for example as sensors operating under high pressures or at high vacuum (analogous to thermal bi-material sensors [24–25]), or, in the construction of systems exhibiting negative compressibility.

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